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# Product quality, product price, and share dynamics in the German compact car market

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The present article examines one of the central elements of evolutionary thinking—competition formalized by the replicator dynamics mechanism. Using data on product characteristics of automobiles sold on the German domestic market over the period 2001–2006, we construct a competitiveness or fitness indicator for each car model applying non-parametric efficiency measurement techniques. The basic question we intend to answer is whether cars providing a higher quality–price ratio for consumers tend to increase their market share compared to variants with lower quality–price ratios. The relationship between a car models' fitness and its market performance is empirically tested in a regression framework. The results show that the principle of "growth of the fitter" is working as suggested by evolutionary theory. In particular, we find that car models with considerably lower fitness than the market average lose additional market shares, whereas models with above-average fitness gain additional market shares.

**JEL classification:** O12, D12, L15.

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## 1. Introduction

Since the seminal work of Nelson and Winter (1982), a multitude of methodological advancements have extended and deepened and the theoretical understanding of the mechanisms of economic evolution (e.g., Dopfer, 2001; Foster and Metcalfe, 2001). One of the fundamental principles underlying evolutionary modeling is competition in general and the principle of "growth of the fitter" in particular (see Nelson and Winter, 1982; Metcalfe, 1994, 1998; Silverberg *et al.*, 1988; Winter *et al.*, 2000, 2003). The principle dates back to Fisher (1930) and can compactly be described by the mechanism of replicator dynamics, stating:

$$\dot{s}_{it} = \frac{ds_{it}}{dt} = \lambda s_{it}\left(f_{it} - \bar{f}_t\right) \qquad i = 1,\ldots,n \qquad t = 1,\ldots,T \tag{1}$$

where $\dot{s}_{it}$ stands for the market share change of firm $i$, $\lambda > 0$ is a parameter controlling the speed of selection, $s_{it}$ denotes the period $t$ market share of a firm $i$ within a population of $n$ competing firms, $f_{it}$ describes the fitness of firm $i$ in period $t$, and $\bar{f}_t = \sum s_{it} f_{it}$ is the share-weighted average fitness in the population. Replicator dynamics implies that firms tend to grow or decline in terms of market shares depending on whether their fitness is above or below the average fitness of all other competing firms in the market. The formulation in (1) obviously represents an ideal setting: demand side effects are entirely represented by the fitness variable $f_{it}$, the market share is a measure of relative size of a firm, where the firms are considered one-product firms, competing in the same, clearly identified and defined market. Despite its simplicity and elegance, when the basic mechanism is applied for explaining the development of certain markets or entire economies, models of high complexity, not permitting analytical solutions, are frequently obtained (Cantner, 2009). As a consequence, agent-based simulation modeling has become the main tool in the evolutionary literature (e.g., Kwasnicki and Kwasnicka, 1992; Saviotti and Mani, 1995; Dosi *et al.*, 1995; Marsili, 2001).

In view of the central theoretical position of replicator dynamics in evolutionary economics it is quite astonishing that empirical attempts trying to answer the question of whether market selection is operating as proposed by evolutionary theory are rare. In principle we can distinguish direct and indirect empirical approaches. The former do not directly test a version of (1) above but look at the implicit relationship between variables representing relative economic success on the one hand and fitness related variables on the other. A first type of indirect empirical studies investigates the formal mechanism of replicator dynamics by linking it to the dynamics of the average fitness variable in a market or industry such as the aggregate productivity development (Cantner and Krüger, 2008; Krüger, 2008). The decomposition of the aggregate productivity change allows identifying their driving forces, which are firm-specific productivity changes, market share changes as well as changes due to the entry and exit of firms. Arguing on the industry-level and using a dataset of

German manufacturing firms over the period 1981–1998, Cantner and Krüger (2008) find a weak tendency that above-average productivity firms are selected in favor of below-average productivity firms. This gives support to a market selection process in the way proposed by replicator dynamics. Note, however, that the results need to be interpreted with caution since Krüger (2008), in a follow-up study, could not confirm a consistent statistical significance.

Another type of indirect empirical analysis in this context is interested in differential growth rates of firms—where differential growth rates imply a market share dynamics as suggested in (1). Using a database of Italian manufacturing firms, Bottazzi *et al.* (2008) investigate how profitability and productivity are related to firm growth. Their results show that the overall selection process is only weakly operating in the expected way. In fact, they do not find a significant relationship between profitability (or productivity) and firm growth (see also Dosi, 2005). In a related study on French manufacturing firms, Coad (2007) raises doubts about the validity of the principle of "growth of the fitter". He finds only a minor influence of profits on growth and concludes that evolutionary models should abandon the assumption of a direct relationship. Coad (2010) indeed shows that subsequent firm growth is initiated by employment growth rather than by growth of profits or sales.

Direct approaches attempt to explicitly test equation (1). Although this appears to be trivial, in practice such an analysis is not easily accomplished (Andersen, 2004), since the data requirements are tremendous. A point in case of a direct empirical test is a study by Metcalfe and Calderini (2000), who compute the selection parameter $\lambda$, measuring the speed of selection, for a dataset of the Italian steel industry. However, for the reasons just mentioned, Metcalfe and Calderini cannot convincingly show that an evolutionary process according to replicator dynamics is at work: an industry is not a market but a collection of markets, the firms are multi-product, and the fitness variable is entirely supply-side determined, unit costs of production. In this context, the first problem is, how to define specific markets and to assign firms that are by all means multi-product and not single-product to these markets is demanding in terms of the availability of internal firm data. In our analysis in this article we suggest to overcome this by shifting the analysis from the firm to the product level. Secondly, the data required for the construction of an appropriate fitness variable are another obstacle. For that we also propose a novel approach in this article.

Hence, the main purpose of this article is to contribute to the few studies that deal with direct empirical tests of the replicator dynamics mechanism. The current article is exceptional in that we consider product variants rather than firms to be the primary unit of selection on markets. By this we solve the problem of assigning multi-product firms to specific markets: first, only those activities of a multi-product firm relevant for the market under investigation are considered; and secondly, we account for the (quite frequent) case that a firm serves a market with similar product variants. Furthermore, we put forward a novel approach for the construction of a fitness variable based on the idea that the competitiveness of a product variant depends on the users' perception of its characteristics and its price. In fact, we assume that products with better characteristics and lower prices will be preferred by consumers.[1] Given this conjecture holds, variants offering a higher ratio of user value and price to consumers should have a competitive advantage. According to replicator dynamics this should come along with increasing market shares within a population of competing products. The crucial part of the article is to test this relationship statistically. In order to do so, we construct a fitness variable for each product model offered on the market. The proposed fitness variable is based on the characteristics of a product and will be interpreted as the ratio of product quality to product price. The computation of a product's fitness is carried out by using non-parametric techniques, adopted from the literature of efficiency analysis.

There is some research examining the competitiveness of products by comparing price jointly with quality (e.g., Papahristodoulou, 1997; Fernandez-Castro and Smith, 2002; Lee *et al.*, 2005; among others). These studies describe a product as a point in the price–quality space and construct a frontier that is determined by the products with lowest price and highest quality. The competitiveness of a product is measured by the distance to the frontier and specified by a single index number called product efficiency. The present article is related to these studies with two major differences. First, we employ robust non-parametric methods to compute the efficiency of products. Robust techniques seem to be better suited in this framework since they are less affected by measurement errors and/or outliers in the data. Second, to the best of our knowledge, the product efficiency concept is applied for the first time to test the replicator dynamics mechanism econometrically. This is done by treating the computed efficiency index as a fitness indicator. Subsequently, the fitness indicator is employed as an explanatory variable in a regression to estimate the parameters of the replicator dynamics equation. The proposed methodology is applied to a specific segment of the German automobile market, namely compact cars.

The remainder of this article is structured as follows. After this short introduction, we will introduce our multi-dimensional measure of fitness in Section 2. This is followed by a discussion of non-parametric techniques used to assess the competitiveness of products in Section 3. Section 4 reports the results of the empirical analysis. A summary and discussion of the main limitations of our methodology is offered in Section 5.

[1]For a number of reasons this might not be the case. Consumers may not have the ability to distinguish the quality of goods, or factors such as brand recognition prevent the selection of the "objectively" best products.

## 2. A multi-dimensional measure of fitness

In most analyses following evolutionary principles, the entity that is selected during the process of competition is a firm. Accordingly, a reallocation of market shares is explained by a market selection process operating on firm-specific characteristics. These firm-specific characteristics are assumed to represent the fundamental sources of firms' differential competitiveness, namely the fitness within a population of heterogeneous economic agents. In evolutionary economics (including theoretical analysis and empirical studies), profit rates, productivity measures, unit costs of production, and product price are most frequently used as proxies for firm competitiveness (see e.g., Nelson and Winter, 1982; Metcalfe, 1994; Mazzucato, 1998; Bottazzi *et al.*, 2008).

Firms in reality, however, are predominantly multi-product firms and therefore are not subject to market selection directly. Instead, we claim that individual products of a firm are subject to selection and the aggregation of these multiple selection processes that are taking place in parallel and possibly interrelated in complex ways determines the fitness and the success of the entire firms. As a consequence, we consider products to be the primary entity of selection, which leads to an indirect selection of the producing firm.

According to Lancaster (1966), consumers do not seek a unique commodity of constant quality, but rather try to satisfy a number of wants through the consumption of a good. These multiple wants are satisfied by different product characteristics, and it is these characteristics, not goods themselves, from which the consumers derive utility. As a result, any fitness variable constructed in this kind of evolutionary framework is required to take the characteristics of products explicitly into account.

Based upon Lancaster's work, Saviotti and Metcalfe (1984) introduced the twin characteristics representation of a product technology. Accordingly, a product can be identified by two sets of characteristics. The technical characteristics describe the internal structure of a product, while service characteristics determine the utility for the users during the process of consumption. Since service characteristics cannot be "produced" directly, there is a pattern of mapping between them. The characteristics approach has been used in various applications. Most frequently, it is applied to measure the degree of technological progress (e.g., Gibbons *et al.*, 1982; Saviotti *et al.*, 1982; Dodson, 1985; Saviotti, 1985; Grupp and Hohmeyer, 1986; Grupp, 1994; Grupp and Maital, 2001), and to identify the emergence of product niches and dominant product designs at the industry level (Frenken *et al.*, 1999; Frenken and Leydesdorff, 2000).

In this article, we take the characteristics approach as the basis for assessing the competitiveness of products. Specifically, we measure a product's competitiveness by computing its distance from a frontier that is spanned by those products that attain a maximum level of competitiveness in a multi-dimensional product characteristics space. In the empirical analysis, this distance from the frontier is used as a proxy for the fitness of a product model. The next section discusses how to derive such a distance measure.

## 3. DEA as a method to assess the performance of products

Data Envelopment Analysis (DEA) is a linear programming procedure to measure the efficiency of observations on the basis of multiple inputs and multiple outputs. The efficiency level of an observation indicates its relative performance and is obtained by comparing an observation to a set of best practice units that shape a so-called efficiency frontier (Cooper *et al.*, 2007).

Another concept to efficiency measurement frequently applied in the literature is the stochastic frontier approach (SFA).[2] The SFA is an econometric estimation technique introduced independently by Aigner *et al.* (1977) and Meeusen and van den Broeck (1977). Compared to most non-parametric approaches, the SFA has the advantage in handling measurement errors and random influences on efficiency. Due to its parametric nature, however, an *a priori* assumption about the shape of the efficiency frontier is required.

The DEA has its origin in the seminal work of Charnes *et al.* (1978) and Banker *et al.* (1984).[3] The basic idea of the DEA is to compare input–output combinations of *Decision Making Units* (DMUs) and to assess their relative performance. Relative performance in terms of efficiency is measured by the distance of DMUs (e.g., firms, products, etc.) to a piecewise empirical extremal production surface that represents the best practice production function. DEA models have a number of attractive properties. DEA approaches, for instance, do not assume that all DMUs have an identical production function. The parametrization of the aggregation functions and thus the aggregation weights are determined endogenously. Moreover, the possibility of using multiple inputs and outputs at the same time is a major advantage of DEA over SFA.

The central idea of the current article is to employ the concept of non-parametric efficiency measurement to assess the competitiveness of products. In fact, we assume that consumers do not search for products with maximum quality or minimum price, but seek to optimize on the quality–price ratio. If we perceive the quality of a product $i$ at time $t$ as a linear combination of $J$ product characteristics $q_{itj}$ ($j = 1, \ldots, J$), collected together in a vector $\mathbf{q}_{it} = (q_{it1}, \ldots, q_{itJ})'$, and denote the

[2]See Kalirajan and Shand (1999) for a detailed comparison between SFA and DEA techniques.

[3]See Charnes *et al.* (2000) and Cooper *et al.* (2007) for an overview about various applications of the DEA concept.

product price $p_{it}$, the ratio between product quality and product price can be formalized as

$$e_{it} = \frac{a_1 q_{it1} + \ldots + a_J q_{itJ}}{p_{it}} = \frac{\mathbf{a}' \mathbf{q}_{it}}{b p_{it}} \tag{2}$$

where the vector **a** contains the weights for aggregating the product characteristics into the scalar product quality measure; $b$ serves as a normalizing constant.

The fitness measure $e_{it}$ is larger if one or several of the measures for the product characteristics are larger at a given price or if the price is smaller for a given bundle of product characteristics. Thus, the fitness measure is analogous to a productivity index, generally defined as a ratio of an output aggregate to an input aggregate. Here, the output is what the consumer receives in terms of services from buying the product and the single input is the price he has to pay. This close resemblance justifies the application of methods for productivity analysis to derive a fitness variable which captures the competitiveness of a product in price–quality space. In Appendix A.1, an output oriented variant of the DEA approach is used to describe the way in which to construct such a fitness measure.

The idea of using non-parametric concepts to quantify the performance of products has already gained interest in the literature. In business economics, DEA is frequently applied to derive market segmentations and to reveal competitive relationships among producers (Despotis *et al.*, 2001; Bauer *et al.*, 2003; Staat and Hammerschmidt, 2005). In engineering, DEA is used as a tool to measure the performance of machines and devices (e.g., Khouja, 1995; Sun, 2002; Triantis, 2003). Also, scholars of the economics of innovation and industrial economics recently employed non-parametric concepts for their purposes (Bernard *et al.*, 1996; Bonaccors *et al.*, 2005; Haller and Grupp, 2009).

The method presented in Appendix A.1 exhibits a severe drawback that is common to all standard DEA models. In fact, every deviation from the frontier is considered as inefficiency. Statistical noise or measurement errors are not accounted for. This makes the approach very sensitive to extreme data points and outliers. In order to overcome these limitations, the order-$m$ approach to robust stochastic non-parametric efficiency measurement is applied here. The basic idea of order-$m$ has been proposed by Cazals *et al.* (2002) and was developed further and applied to real data by Daraio and Simar (2005, 2007a,b), Simar (2007), Simar and Zelenyuk (2008) and Wheelock and Wilson (2004). The application of the order-$m$ method on a product dataset implies that the efficiency of each product is evaluated repeatedly against a partial product-efficiency frontier spanned by $m$ of the sample products. This gives an estimator of our fitness variable that is quite robust to outliers and measurement errors. Details can be found in the working paper version of this article.[4]

[4]Cantner *et al.* (2010).

# 4. Empirical analysis

## 4.1 Data description and sample selection

The subject of the empirical analysis is the segment of compact cars in the German market. We employ two distinct data sources to obtain the required information. Sales data are available from the Kraftfahrtbundesamt[5] (KBA), Germany's national road vehicle registration authority. The KBA annually publishes data on the frequency of sales of specific car models in the "New registrations for motor vehicle and vehicle trailer by type, size class, producer and federal state"[6] statistics. To ensure a rather homogeneous dataset and in order to avoid a comparison of "apples and oranges", we restrict our analysis to a particular segment of the entire car market, namely the market for compact cars. To distinguish compact cars from non-compact cars, standard classifications offered by the KBA are used.[7]

Information on prices and quality attributes for each car model come from the ADAC, Germany's largest automobile club. The ADAC annually publishes electronic lists containing data on prices, and technical and non-technical features of new cars. These electronic databases are used to collect data on quality attributes of cars. Altogether, information on 41 quality attributes for each variant of the numerous car models is accessible.[8] Note that the price information for new cars does not incorporate any sales returns or rebates which are quite often used in car purchasing. However, in the absence of more detailed price information, we assume that the list price is the most reliable proxy variable available.

Information on sales frequency is provided on the level of car models (e.g., VW Golf 1.6). Since the ADAC database offers data on prices and quality features for specific variants (e.g., VW Golf 1.6 Trendline), we proceed by aggregating price and quality data for the various variants of the same model. In fact, we calculate the arithmetic mean for each attribute over all variants of the same car model. This yields a vector of average quality attributes (including price) for each car model.

[5]Federal Motor Transport Authority.

[6]Statistics for "Neuzulassungen von Kraftfahrzeugen und Kraftfahrzeuganhängern nach Fahrzeugarten, Größenklassen, Herstellern, Typen und Bundesländern."

[7]Note that the KBA does not provide detailed information as to how such a classification is derived, i.e., what kind of technical specification (size, engine power, etc.) is required to identify a car as a compact.

[8]A model variant is a specific version of a car model that differs from others version of the same model by a few attributes such as the availability of optional items (supplementary equipment). The main characteristics such as type of engine, horsepower, or cylinder capacity are the same for each variant of the same model. The terminology used here is described as follows: a brand would be "VW," a product line would be "VW Golf," a model would be "VW Golf 1.6," and a specific variant would be "VW Golf 1.6 Trendline."

### *4.2 The German car market*

In 2007, Germany accounted for almost 11% of the worldwide automobile production (OICA, 2008). The German automotive industry is one of the major backbones of the German economy. With a strong labor force of around 745,000 the industry is one of the most important employers in the country (VDA, 2008). Germany is the largest national car market in Europe (ACEA, 2008). Between 1998 and 2007, on average, 3.43 million passenger cars per year were sold (KBA, 2008a). The total vehicle population is of about 41 million passenger cars (KBA, 2008b). Domestic brands appear to have a competitive advantage. According to statistics of the KBA (2008a), in 2007, 64% of new registered cars were produced by German manufacturers. Figure 1 illustrates the market share development of the largest brands in Germany. We see that VW (Volkswagen) is the undisputed market leader, followed by Mercedes, Opel, BMW, and Audi. Ford, ranked at the sixth place, is the first foreign brand among the leading automobile manufacturers.

According to the KBA, the entire car market is divided into 10 segments. The segments are defined in terms of horsepower, cylinder capacity, size, design, and price. During the period under observation, the demand structure changed considerably (Figure 2). In 2001, cars of the middle class attracted 25% of new car buyers; small cars realized a market share of 17%. In subsequent years, small cars increased their market share to 19%, while middle class cars lost market share considerably. In 2007, the middle class accounted for only 16% of the entire market. Also the demand for vans exhibited a remarkable change. This segment increased its market penetration from 7% in 2001 to 12% in 2007. The market share of all other segments remained fairly stable over the time span. Figure 2 clearly indicates that the compact class is the largest segment in the German car market. Approximately 26% of all newly registered cars in Germany were compact cars.

We were able to collect reliable information on sales of compact cars for the period 2001–2006. Table 1 shows the number of car models per manufacturer within our data sample.[9] From the table we see that the average number of car models has increased over time. In addition, we observe considerable differences across producers and these differences are quite stable over the sample period.

### *4.3 Car efficiency estimates*

In order to compute the efficiency of a car model—indicating its fitness—the order-*m* approach is applied to the data. The first step required is a consideration of inputs and outputs to be used in the analysis. The choice of the "right" characteristics is a crucial task, as it determines the accuracy of later statistical analyses. We have already

[9]The table does not contain the number compact car models in 2006. Since we analyze the relationship between fitness and market share changes it is not necessary to report this information for the year 2006.

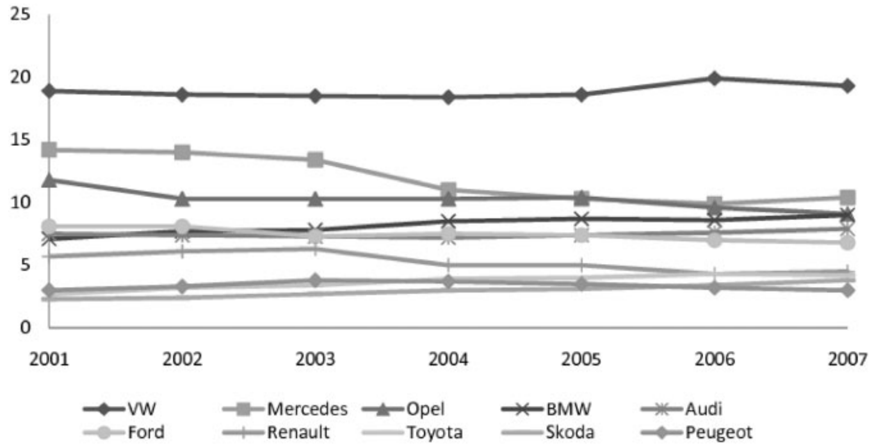


**Figure 1.** Market share development of the 10 largest brands in Germany between 2001 and 2007. *Source*: KBA (press releases on passenger car registrations, 2001–2007).

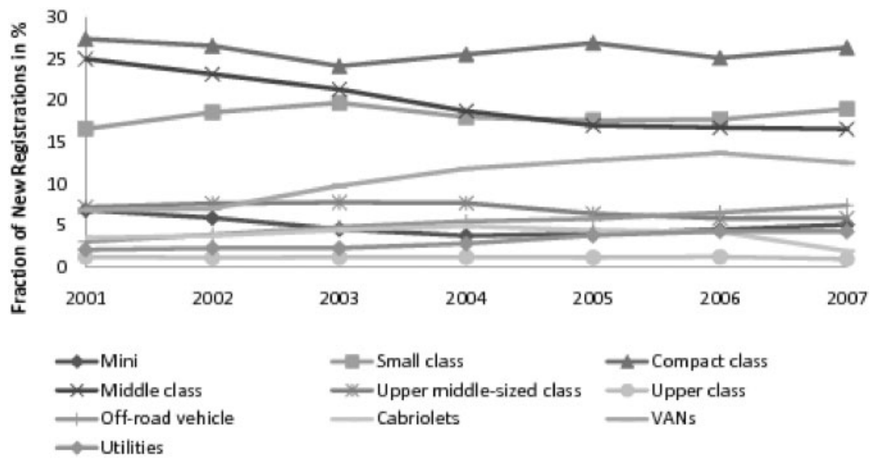


**Figure 2.** Fraction of sales by segments between 2001 and 2007. *Source*: KBA (press releases on passenger car registrations, 2001–2007).

pointed out that consumers are primarily interested in the services delivered by a product. However, since service characteristics cannot be produced "directly," producers need to modify technical characteristics in order to enhance a products' service characteristics. In the case of a car, typical service characteristics influencing the evaluation of potential car buyers are speed of transportation, ecological efficiency, safety, space, convenience, etc. Corresponding technical characteristics are

**Table 1** Number of compact car models per manufacturer

| Producer | Year 2001 | 2002 | 2003 | 2004 | 2005 |
|---|---|---|---|---|---|
| Audi | 13 | 12 | 16 | 10 | 10 |
| Chrysler | 2 | 2 | 3 | 0 | 0 |
| Citroen | 16 | 13 | 14 | 18 | 25 |
| Daewoo | 7 | 0 | 2 | 7 | 10 |
| Daimler | 8 | 8 | 7 | 13 | 11 |
| Fiat | 28 | 16 | 17 | 23 | 34 |
| Ford | 20 | 26 | 26 | 34 | 40 |
| Honda | 15 | 9 | 8 | 9 | 11 |
| Hyundai | 3 | 4 | 5 | 4 | 3 |
| Kia | 8 | 7 | 7 | 11 | 13 |
| Lada | 5 | 3 | 3 | 6 | 8 |
| Mazda | 11 | 8 | 12 | 7 | 7 |
| Mitsubishi | 8 | 6 | 9 | 11 | 12 |
| Nissan | 5 | 8 | 8 | 6 | 8 |
| Opel | 36 | 33 | 35 | 51 | 43 |
| Peugeot | 17 | 15 | 12 | 15 | 16 |
| Renault | 18 | 21 | 26 | 21 | 31 |
| Rover | 9 | 9 | 9 | 9 | 7 |
| Seat | 18 | 19 | 21 | 16 | 21 |
| Skoda | 18 | 17 | 19 | 27 | 30 |
| Subaru | 6 | 6 | 5 | 5 | 10 |
| Suzuki | 11 | 11 | 10 | 8 | 7 |
| Toyota | 15 | 20 | 15 | 20 | 18 |
| VW | 78 | 76 | 76 | 62 | 50 |
| Total | 375 | 349 | 365 | 393 | 425 |

engine power, fuel consumption, number of airbags, dimension, and special equipment available, to mention just a few.

Since cars are highly differentiated products, the full range of characteristics to distinguish one car model from another is very large (Papahristodoulou, 1997). To guarantee a correct efficiency evaluation, ideally all of them should be taken into account. However, various quality attributes are not measurable in an objective way (e.g., style) or necessary data on specific characteristics are not accessible. Faced with that problem, we restrict our analysis to a small subset of the possible characteristics. In order to ensure that the selected characteristics are relevant for the buying decision of car buyers, we use expert judgments gathered by means of interviews, questionnaires, and other types of corresponding publications. In particular, we apply only those characteristics that are frequently regarded as important by consumer reports or related studies (e.g., Staat *et al*., 2002; Oh *et al*., 2005; DAT, 2006; ADAC, 2007). To avoid the use of redundant information, we conduct a correlation test among the relevant characteristics (see Table A.1 in the Appendix).

In the end, the following technical characteristics are incorporated in the efficiency measurement of a car model. The maximum engine power in kilowatts is used as a proxy for the performance of a car. Fuel efficiency, defined as the amount of covered distance (in kilometers) per liter of petrol, indicates the environmental friendliness of a car model and is obtained by calculating the reciprocal of fuel consumption. As an indicator for the loading capacity, we utilize the luggage space (in liters), and as a proxy for safety, the dimension (length$\times$width$\times$height) of a car in cubic meters is employed.[10] The list price of a car model specifies the cost parameter. Basic descriptive statistics of the characteristics incorporated in the annual order-$m$ estimation are reported in Table 2.

The second step of the empirical analysis is the computation of order-$m$ efficiency estimates. The four technical performance characteristics serve as outputs in the non-parametric frontier estimation approach. As sole input variable, the list price of a car model is used. For the purpose of this study, efficiency is measured in output orientation. The order-$m$ scores are computed using the package "FEAR" for R, supplied by Paul Wilson on his web page (see Wilson, 2008). Note that, in contrast to standard DEA approaches, the order-$m$ estimates are not bounded by 1.[11] As the number of car models is very large, Table 3 illustrates the summary statistics of efficiency estimates for different years.

Table 3 reveals a remarkable degree of stationarity. Minimum, maximum, median, and mean efficiency do not exhibit remarkable changes over time. Additionally, we provide Gaussian Kernel density estimates to display the distribution of the efficiency measures over time (Figure 3). The Kernel bandwidth is chosen according to Silverman's rule of thumb (Silverman, 1986). Visual inspection of this plot displays two distinct features. First, the distribution of efficiency scores remains nearly constant over time. Second, the right tails of the efficiency distributions in Figure 3 indicate the existence of a number of car models with relatively high efficiency estimates compared with the rest of the market.

[10]As an alternative to the dimension as a proxy for safety, the number of airbags can be applied. However, until the year 2004, we had only information about the endowment with driver-, passenger-, and side-airbag. Since airbags became more and more a standard feature during the period of investigation, the variation in the number of airbags declined sharply. To avoid problems caused by this low variation, we follow Papahristodoulou (1997) and rely on the dimension as a proxy for safety. Note, however, that the efficiency scores calculated using the number of airbags do not substantially differ.

[11]The reason for this is the allowance for random noise.

**Table 2** Descriptive statistics of characteristics incorporated in the order-$m$ estimation (by year)

| | Characteristic | Mean (SD) | Min | Max | Skewn. | Kurt. |
|---|---|---|---|---|---|---|
| | Kilowatt | 80 (21) | 44 | 160 | 1.29 | 5.26 |
| | Fuel Efficiency | 14.90 (2.94) | 9.09 | 22.22 | 0.46 | 3.09 |
| 2001 | Luggage | 400 (81) | 177 | 550 | −0.21 | 2.46 |
| | Dimension | 10.54 (0.61) | 9.02 | 12.26 | −0.05 | 2.97 |
| | Price | 18292 (3825) | 9965 | 30350 | 0.64 | 3.06 |
| | Kilowatt | 84 (23) | 44 | 195 | 1.32 | 5.21 |
| | Fuel Efficiency | 15.03 (2.93) | 8.26 | 22.22 | 0.24 | 2.14 |
| 2002 | Luggage | 395 (83) | 177 | 550 | −0.07 | 2.24 |
| | Dimension | 10.72 (0.57) | 9.19 | 12.26 | 0.09 | 2.87 |
| | Price | 19475 (3918) | 10890 | 37340 | 0.74 | 3.85 |
| | Kilowatt | 86 (25) | 50 | 195 | 1.39 | 5.33 |
| | Fuel Efficiency | 15.06 (2.99) | 8.26 | 22.22 | 0.26 | 2.22 |
| 2003 | Luggage | 393 (81) | 209 | 550 | 0.16 | 2.18 |
| | Dimension | 10.81 (0.55) | 9.20 | 12.45 | 0.50 | 3.11 |
| | Price | 19789 (3857) | 11445 | 37340 | 0.79 | 4.04 |
| | Kilowatt | 86 (25) | 50 | 195 | 1.74 | 7.00 |
| | Fuel Efficiency | 15.31 (3.02) | 8.06 | 22.22 | 0.25 | 2.25 |
| 2004 | Luggage | 402 (83) | 209 | 560 | 0.17 | 2.11 |
| | Dimension | 10.94 (0.56) | 9.76 | 12.50 | 0.45 | 3.06 |
| | Price | 19788 (3775) | 11445 | 38490 | 1.38 | 7.00 |
| | Kilowatt | 87 (24) | 50 | 195 | 1.66 | 6.59 |
| | Fuel Efficiency | 15.78 (3.06) | 8.06 | 22.22 | 0.26 | 2.23 |
| 2005 | Luggage | 408 (91) | 209 | 580 | 0.27 | 1.95 |
| | Dimension | 11.18 (0.62) | 8.55 | 12.50 | −0.08 | 3.43 |
| | Price | 20008 (3706) | 7633 | 38490 | 0.86 | 6.93 |

**Table 3** Summary statistics of efficiency estimates

| Year | Min | 1st Quan. | Median | Mean | 3rd Quan. | Max |
|---|---|---|---|---|---|---|
| 2001 | 0.919 | 1.017 | 1.059 | 1.068 | 1.107 | 1.323 |
| 2002 | 0.941 | 1.022 | 1.062 | 1.074 | 1.113 | 1.401 |
| 2003 | 0.947 | 1.020 | 1.061 | 1.074 | 1.110 | 1.405 |
| 2004 | 0.938 | 1.016 | 1.061 | 1.073 | 1.105 | 1.364 |
| 2005 | 0.913 | 1.011 | 1.048 | 1.066 | 1.095 | 1.375 |

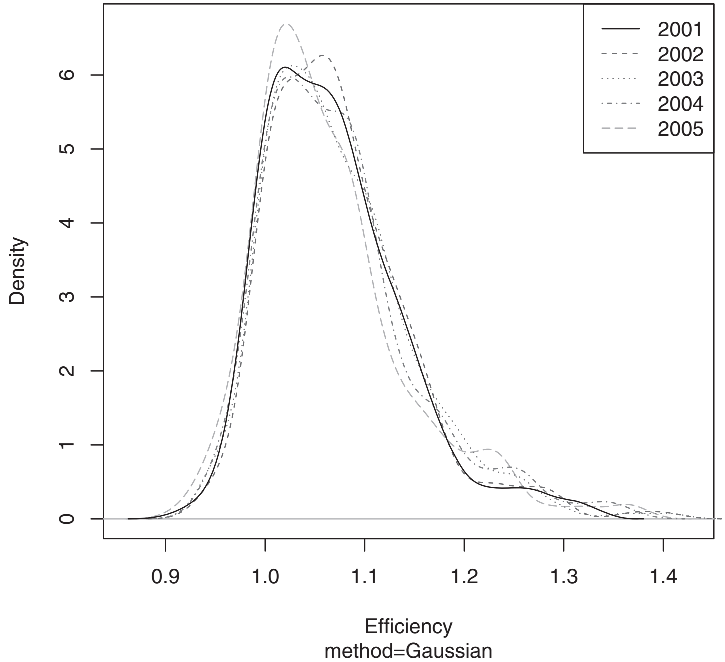


**Figure 3** Kernel density estimates of car efficiency scores 2001–2005.

## *4.4 Car fitness and market success*

In this section, we analyze the relationship between performance superiority and market success. As described above, efficiency is assumed to indicate the competitiveness of a car in quality–price space. Now, we employ this efficiency measure as the fitness variable in the replicator dynamics equation. According to the "growth of the fitter" principle, we suspect above-average fitness to be associated with growth in terms of market shares at the product level.

The replicator dynamics mechanism is the same as in equation (1) with the fitness variable defined in (2). The replicator dynamics equation in discrete time is given by

$$\triangle s_{i,t:t+k} = s_{i,t+k} - s_{it} = \lambda s_{it}(e_{it} - \overline{e}_t) \tag{3}$$

where $s_{it}$ stands for the market share of car model $i$ within a population of $n$ competing car models, $e_{it}$ denotes the fitness of a certain model, and $\bar{e}_t = \sum s_{it}e_{it}$ is the (share-weighted) average fitness on the market.

In order to estimate (3), we transform the equation into the following econometric model:

$$\triangle s_{i,t:t+k} = \beta_1 FIND_{it} + \gamma' x_{it} + u_{it}, \qquad t = 2001,\ldots,2005 \qquad k = 1,\ldots,5 \tag{4}$$

where we define $FIND_{it} = s_{it}(e_{it} - \overline{e}_t)$ and label this variable as the fitness indicator. The respective parameter to be estimated is $\beta_1$ and $u_{it}$ is an i.i.d. error term.[12] The dependent variable, $\triangle s_{i,t:t+k}$, denotes the change in market share of car model $i$ between period $t$ and $t+k$ within a cohort of competing car models on the market at time $t$. In fact, $FIND_{it}$ is the market share weighted deviation of a car model from the average fitness in the market. From the theoretical considerations described above, we expect a positive sign of the estimated coefficient $\beta_1$. A positive coefficient implies that car models with an above-average fitness at time $t$ tend to increase their market share between $t$ and $t+k$, while models with a below-average fitness lose in terms of market shares. The term $\boldsymbol{x}_{it} = (Age_{it}, VW_i, Opel_i, Ford_i, \ldots, Peugeot_i, Year\ Dummies_t)'$ stands for a vector of control variables that enter the empirical analysis.

The first step in our econometric analysis is to estimate equation (4) using ordinary least squares regressions (OLS). A Breusch–Pagan test reveals a heteroscedastic nature of the data. A potential source for heteroscedasticity that violates the i.i.d. assumption of the OLS estimator could be the correlation of market shares of car models within the same product line. In order to rule out misleading statistical inference we use heteroscedasticity-consistent standard errors as proposed by White (1980).

Table 4 presents the regression results estimating equation (4) for various years and changing time lag $k$ of the dependent variable.

The initial model solely incorporates the fitness indicator of model $i$ at time $t$ ($FIND$). Other control variables have not been included since we want to estimate the replicator dynamics equation in its pure form. A closer look at the regression results in Table 4 reveals that the fitness indicator of car models in a base year only partly explains the market share development in subsequent years. The estimates for the group of regressions with $k=1$, i.e., when the independent variable is the change in market share between $t$ and $t+1$, do not point to a significant effect of a product's fitness indicator on the market share development. Only for $t=2003$ a significantly positive coefficient is obtained. However, increasing the parameter $k$ makes us more confident that our fitness indicator works in the expected way. Except for the parameter constellation $k=2$ and $t=2001$, the signs of the estimated coefficients are always positive. This suggests that car models with a higher competitiveness and thus providing a better quality–price ratio to the consumers were able to increase their market share. However, even if most of the estimates turn out to be positive, not all of the corresponding coefficients are significant. Moreover, an unstable and sometimes fairly small R-squared does not reflect a good fit of this simple model. Obviously, other unobserved factors heavily influence the market performance of automobiles. For instance, we could think of brand effects that impinge on sales.

[12] $\beta_1$ can be interpreted as a parameter that accounts for the speed of selection. It is equivalent to the parameter $\lambda$ in equation (3).

**Table 4** OLS regression results for 2001–2005

| | | | | | |
|---|---|---|---|---|---|
| $k=1$ | $t=2001$ | $t=2002$ | $t=2003$ | $t=2004$ | $t=2005$ |
| FIND | −1.3813 | −0.1921 | 10.1254** | −3.0383 | −1.526 |
| | (0.8603) | (0.72964) | (4.2198) | (2.7386) | (3.3214) |
| $R^2$ | 0.01864 | 0.3892 | 0.3255 | 0.05667 | 0.01718 |
| Obs. | 326 | 286 | 300 | 313 | 336 |
| $k=2$ | $t=2001$ | $t=2002$ | $t=2003$ | $t=2004$ | |
| FIND | −2.6381** | 10.6850** | 13.4381*** | 9.2037** | |
| | (1.1093) | (5.3073) | (5.0123) | (3.7434) | |
| $R^2$ | 0.03794 | 0.2551 | 0.1866 | 0.08587 | |
| Obs. | 326 | 286 | 300 | 313 | |
| $k=3$ | $t=2001$ | $t=2002$ | $t=2003$ | | |
| FIND | 6.9514 | 12.9329** | 10.6517* | | |
| | (6.4364) | (5.1055) | (5.5161) | | |
| $R^2$ | 0.08511 | 0.1642 | 0.06322 | | |
| Obs. | 326 | 286 | 300 | | |
| $k=4$ | $t=2001$ | $t=2002$ | | | |
| FIND | 9.7523* | 14.6005** | | | |
| | (5.3163) | (7.3995) | | | |
| $R^2$ | 0.06845 | 0.08308 | | | |
| Obs. | 326 | 286 | | | |
| $k=5$ | $t=2001$ | | | | |
| FIND | 8.4671 | | | | |
| | (5.7581) | | | | |
| $R^2$ | 0.0155 | | | | |
| Obs. | 326 | | | | |

*Notes*: Dependent variable is the market share change between $t$ and $t+k$. Robust standard errors are reported in parentheses. ***1% level of significance, **5% level of significance, *10% level of significance.

The impact of brand image in the automobile market is subject of a number of studies (De Pelsmacker, 1988; Mannering and Winston, 1991; Nichols, 1998; among others). This literature stresses that the brand image is a key element for the long-term success on the market. Swait (1994) argues that the impact of brand image on the buying decision becomes even stronger when costumers imperfectly observe the attributes of products. With respect to automobiles, this is certainly the case. Nevertheless, some preliminary conclusions from the initial model can already be drawn: (i) in the short-run, i.e., with a lag of one year in the dependent variable, no clear effect of a car model's fitness indicator on its market performance can be monitored; (ii) in the longer run, a product's fitness indicator positively affects market share growth; (iii) factors other than fitness apparently determine the economic performance of car models. In the following, we check whether these preliminary findings turn out to be robust across alternative specifications of the basic model.

In the next step of our analysis we move from yearly to pooled OLS regressions. Pooling yearly cross-sections increases the sample size and provides more powerful test statistics with respect to statistical inference. In order to account for brand specific factors, we include dummy variables for the 10 largest compact car producers in all our regressions.[13] Further, year dummies enter the estimations (baseline year = 2001). The pooled OLS estimates are displayed in Table 5. In order to ease the interpretation of coefficient magnitudes the regressions are calculated with standardized data.[14]

With regard to the relationship between the car model fitness indicator (*FIND*) and market share changes, Table 5 reveals positive signs of the coefficients. This implies that car models with a higher fitness indicator systematically tend to gain market shares, exactly as replicator dynamics suggests. However, as in the case of the yearly OLS, a positive and significant impact on market shares cannot be observed in the short-run (i.e., with lag of one year). Increasing the time lag of the response variable makes us more confident that car fitness works in the expected way. For $k \in \{2, 3, 4\}$, the estimated coefficients are statistically significant, which shows a positive impact of the fitness indicator on changes in market share. Hence, producing car models that offer a high quality–price ratio to customers does not seem to pay off in the short but rather in the longer run.[15] A variable *Age*, accounting for the number of years since market introduction of a car model, is incorporated in the estimation. Since car buyers might prefer car models that are more up to date, the variable reflects the valuation of consumers for modern cars. Another rationale behind the inclusion of this variable is that car models might exhibit a negative growth in market shares due to market exit. This market exit can be the result of a bad economic performance, but it can also be induced by the decision of a manufacturer to stop a model's production in response to the introduction of a successor model. We assume this is more likely to occur for cars that had already been a considerable time on the market. The variable *Age* controls for these effects. The empirical results illustrated in Table 5 are fully in line with our expectation.

[13] All other brands in the market serve as a reference group.

[14] Of course, dummy variables are not standardized.

[15] Analogous regressions with the lagged market share replacing the fitness indicator indeed show reversion-to-the-mean dynamics. Once the fitness indicator is considered, however, this pattern disappears. In fact, the results show that the lagged market share is never significant, but some explanatory power is drawn from the fitness indicator.

**Table 5** Pooled OLS regression results

| | $k=1$ | $k=2$ | $k=3$ | $k=4$ |
|---|---|---|---|---|
| FIND | 0.0521 | 0.2870*** | 0.3257*** | 0.3392*** |
| | (0.1199) | (0.0987) | (0.1005) | (0.1276) |
| Age | −0.1680*** | −0.0822** | −0.0822*** | −0.1363*** |
| | (0.0276) | (0.0337) | (0.0337) | (0.0481) |
| VW | 0.0390 | −0.0512 | 0.0187 | 0.1821 |
| | (0.1001) | (0.0892) | (0.1199) | (0.1945) |
| Opel | −0.2653*** | −0.2634*** | −0.2945*** | −0.3644*** |
| | (0.0677) | (0.0622) | (0.0755) | (0.0986) |
| Ford | 0.0789 | −0.0824 | −0.0751 | −0.3713*** |
| | (0.0859) | (0.0948) | (0.0948) | (0.1198) |
| Daimler | −0.2219 | 0.1261 | 0.0094 | −0.4910 |
| | (0.2615) | (0.3556) | (0.4934) | (0.4317) |
| Audi | 0.6282** | 0.3184 | −0.2581** | −0.2969* |
| | (0.2588) | (0.4032) | (0.1282) | (0.1787) |
| Toyota | −0.0260 | 0.5071** | 0.7460** | 0.6744 |
| | (0.0903) | (0.2182) | (0.3590) | (0.4695) |
| Skoda | 0.2127*** | 0.0496 | 0.1777 | 0.2978 |
| | (0.0682) | (0.0720) | (0.1186) | (0.2012) |
| Citroen | 0.0007 | 0.0021 | 0.0042 | 0.0785 |
| | (0.0421) | (0.0443) | (0.0643) | (0.1245) |
| Renault | −0.1937*** | −0.2111*** | −0.1560*** | −0.0789 |
| | (0.0502) | (0.0515) | (0.0594) | (0.0552) |
| Peugeot | 0.0473 | 0.2410* | 0.2439 | 0.4243* |
| | (0.0738) | (0.1373) | (0.1754) | (0.2494) |
| Year dummies | Yes | Yes | Yes | Yes |
| $R^2$ | 0.053 | 0.118 | 0.132 | 0.122 |
| F-statistic | 5.64 | 5.27 | 5.87 | 5.13 |
| *P*-value | 0.000 | 0.000 | 0.000 | 0.000 |
| Obs. | 1561 | 1225 | 912 | 612 |

*Notes*: Dependent variable is the market share change between $t$ and $t+k$. Robust standard are reported errors in parentheses. ***1% level of significance, **5% level of significance, *10% level of significance.

The coefficients for the variable *Age* are all negative and significant, revealing a negative impact of time since market introduction on share development. As already described, this might reflect the fact that newer car models are more attractive for potential buyers. Alternatively, however, one can argue that older car models are more likely to be substituted by manufacturers. Unfortunately, the dataset does not allow us to disentangle both effects.

The results of Table 5 further suggest that brand-specific factors entail an impact on the market success of car models. The statistical significance of the manufacturer dummies indicates that market share changes might partly be due to idiosyncratic effects related to the car producer. Over the observed time span, in particular, automobiles from Opel and Renault performed poorly in economic terms. To a minor extent the same holds for Audi. Quite the contrary is found for Toyota. Evidently, Toyota was able to meet the taste of German consumers, which kept the market shares of its car models growing. Notably, the inclusion of additional control variables for brand-specific impacts does not substantially change the basic findings. Table 5 shows that the fitness indicator (*FIND*) is an important explanatory variable for market share changes.

Instead of using the deviation from the average fitness in the market, all model specifications were re-estimated merely employing the fitness measure $e_{it}$ of automobiles as an explanatory variable (results not shown). Quite interestingly, the results reveal a positive but never a significant effect of fitness on changes in market share.

Next, we want to present a further robustness check of our preliminary finding that car models with an above-average fitness systematically tend to gain market shares. So far, the order-$m$ approach is used to construct the fitness indicator variable (*FIND*). In the following the same econometric model as in Table 5 is estimated, but now the quantile-based order-$\alpha$ approach is applied to determine a car models" fitness.

The order-$\alpha$ method for the general multiple-input multiple-output case is developed by Daouia and Simar (2007). They show that order-$\alpha$ efficiency estimates have a bounded influence function, while order-$m$ efficiency measures have an unbounded influence function and hence are less robust. The correlation coefficient between the order-$\alpha$ and the order-$m$ estimates is 0.96. Regression results applying the order-$\alpha$ approach can be found in Table 6.[16]

Table 6 confirms the findings of the previous regressions. We find a positive and significant effect of the fitness indicator (*FIND*) on market share changes, at least in the longer run. Compared with Table 5 the magnitudes of the coefficients are slightly larger. The parameter estimates of the variable *Age* as well as the manufacturer dummies keep their signs and the levels of significance. In summary it can be said that the robustness check supports the previous finding of a positive relationship between the fitness indicator and subsequent growth in terms of market shares. This points to a replicator dynamics mechanism operating at the product level.

[16] In this article $\alpha$ is set to 0.95.

**Table 6** Pooled OLS regressions using the order-$\alpha$ approach

| | $k=1$ | $k=2$ | $k=3$ | $k=4$ |
|---|---|---|---|---|
| FIND | 0.0622 | 0.3060*** | 0.3962*** | 0.3991*** |
| | (0.1180) | (0.1095) | (0.0979) | (0.1313) |
| Age | −0.1678*** | −0.0816** | −0.0729** | −0.1333*** |
| | (0.0276) | (0.0332) | (0.0326) | (0.0488) |
| VW | 0.0422 | −0.0456 | 0.0316 | 0.1985 |
| | (0.1018) | (0.0891) | (0.1171) | (0.1947) |
| Opel | −0.2682*** | −0.2840*** | −0.3166*** | −0.4215*** |
| | (0.0676) | (0.0652) | (0.0872) | (0.1218) |
| Ford | 0.0778 | −0.0803 | −0.0887 | −0.3920*** |
| | (0.0861) | (0.0966) | (0.1358) | (0.1288) |
| Daimler | −0.2208 | 0.1151 | 0.0535 | −0.4662 |
| | (0.2561) | (0.3413) | (0.4725) | (0.3685) |
| Audi | 0.6229** | 0.2982 | −0.2601** | −0.2683 |
| | (0.2533) | (0.3932) | (0.1207) | (0.1670) |
| Toyota | −0.0254 | 0.5068** | 0.7522** | 0.6723 |
| | (0.0900) | (0.2182) | (0.3609) | (0.4702) |
| Skoda | 0.2098*** | 0.0481 | 0.1478 | 0.2723 |
| | (0.0668) | (0.0736) | (0.1180) | (0.2040) |
| Citroen | 0.0002 | 0.0018 | −0.0086 | 0.0675 |
| | (0.0418) | (0.0461) | (0.0633) | (0.1229) |
| Renault | −0.1950*** | −0.2146*** | −0.1629** | −0.0742 |
| | (0.0509) | (0.0536) | (0.0648) | (0.0536) |
| Peugeot | 0.0476 | 0.2458* | 0.2411 | 0.4226* |
| | (0.0730) | (0.1383) | (0.1747) | (0.2472) |
| Year dummies | yes | yes | yes | yes |
| $R^2$ | 0.054 | 0.130 | 0.169 | 0.148 |
| F-statistic | 5.55 | 5.24 | 6.75 | 5.73 |
| $P$-value | 0.000 | 0.000 | 0.000 | 0.000 |
| Obs. | 1555 | 1219 | 906 | 606 |

*Notes*: Dependent variable is the market share change between $t$ and $t+k$. Robust standard are reported errors in parentheses. ***1% level of significance, **5% level of significance, *10% level of significance.

As additional robustness checks we investigate whether the selection process for German car manufacturer is different from the rest and whether there are differences between the subsamples of low- and high-price cars. The results can be found in Tables 7 and 8.

**Table 7** Testing the selection mechanism for German and non-German cars

| | $k=1$ | $k=2$ | $k=3$ | $k=4$ |
|---|---|---|---|---|
| FIND | 0.0658 | −0.0949 | −0.2404 | −0.2537 |
| | (0.0981) | (0.1135) | (0.1515) | (0.1838) |
| German·FIND | 0.0178 | 0.4616*** | 0.6654*** | 0.6969*** |
| | (0.1693) | (0.1613) | (0.1867) | (0.2292) |
| Year dummies | Yes | Yes | Yes | Yes |
| $R^2$ | 0.007 | 0.111 | 0.137 | 0.108 |
| *F*-statistic | 0.15 | 3.45 | 5.19 | 4.85 |
| *P*-value | 0.988 | 0.004 | 0.000 | 0.002 |
| Obs. | 1561 | 1225 | 912 | 612 |

*Notes*: Dependent variable is the market share change between $t$ and $t+k$. Robust standard are reported errors in parentheses. ***1% level of significance, **5% level of significance, *10% level of significance.

In Table 7, we test whether the mechanism is different for German car brands compared to the entire sample. Therefore, we introduce an interaction term of *FIND* and a German car dummy variable. We find the associated coefficient significantly positive and considerable in magnitude except for the lowest $k$. Simultaneously the *FIND* variable itself becomes insignificant. This implies that the main results of the article are driven by the German car brands that make up 32% of the sample.

Table 8 reports the regression results for the subsamples of low-price (with below-median price) and high-price (with above-median price) cars and $k=4$. We observe considerable differences for the two subsamples. Only in the low-price subsample we find evidence for the replicator dynamics at work. The *FIND* variable in strongly significant, has positive sign and is of considerable magnitude here. In addition, the dummy variables for the different car manufacturer have the same structure as in Table 5 above, albeit with a much better fit here. For the high-price subsample other considerations of the customers than those taken as characteristics for our measurement appear to play a major role. Here, the *FIND* variable is insignificant and the significance of the coefficient estimates is generally lower than for the low-price sample. In addition, the overall fit measured by $R^2$ is much lower in the case of the high-price subsample.

## 5. Conclusions

The present article aims at shedding light on the empirical validation of the principle of 'growth of the fitter' as a central element of evolutionary thinking. In particular, we explore the relevance of the replicator dynamics mechanism in the German

**Table 8** Testing the selection mechanism for low- and high-price cars

| | Low-price | High-price |
|---|---|---|
| FIND | 0.3477*** | 0.3939 |
| | (0.1060) | (0.3229) |
| Age | −0.1189** | −0.1645** |
| | (0.0544) | (0.0825) |
| VW | 0.0100 | 0.2432 |
| | (0.4252) | (0.1979) |
| Opel | −0.5570*** | −0.1974* |
| | (0.1829) | (0.1117) |
| Ford | −0.3233*** | −0.5519 |
| | (0.1101) | (0.3420) |
| Daimler | 0.4853 | −0.8953* |
| | (0.5376) | (0.5173) |
| Audi | – | −0.2461 |
| | – | (0.2329) |
| Toyota | 0.8617 | −0.1801 |
| | (0.5801) | (0.1463) |
| Skoda | 0.7965 | 0.1333 |
| | (0.5269) | (0.2220) |
| Citroen | 0.0219 | 0.1100 |
| | (0.1558) | (0.1854) |
| Renault | −0.1239* | −0.1287 |
| | (0.0639) | (0.1487) |
| Peugeot | 0.2787 | 0.7008 |
| | (0.1972) | (0.7347) |
| Year dummies | yes | yes |
| $R^2$ | 0.243 | 0.082 |
| $F$-statistic | 5.90 | 1.90 |
| $P$-value | 0.000 | 0.030 |
| Obs. | 306 | 306 |

*Notes*: Dependent variable is the market share change between $t$ and $t+k$. Robust standard are reported errors in parentheses. ***1% level of significance, **5% level of significance, *10% level of significance.

compact car market. Unlike most empirical analyses in an evolutionary framework, our approach considers products, namely car models, to be the primary units of selection on markets. Based on product characteristics, we calculate the fitness for each automobile by employing a stochastic version of a non-parametric efficiency measurement approach. The fitness indicator is used to estimate the replicator dynamics equation econometrically. Our results provide evidence for the existence of a market selection process according to the replicator dynamics mechanism. Indeed, we find that car models with considerably lower fitness than the market average lose market shares, while models with above-average fitness gain additional market shares.

In view of these results our article contributes to the direct empirical analysis of the force of the replicator dynamics, so prominent in theorizing in an evolutionary economics context. To our knowledge this is the first analysis directly estimating the selection equation. Former attempts, as referred to in the introduction, to validate this dynamics have been much less successful and found almost no significant indications. A most likely reason for this seems to be that there the level of analysis has always been an industry or a broader sector. Taking replicator dynamics literally, however, it is the market level that has to be addressed and hence the competition among products instead of firms. Our approach just attempts to improve on that by putting the replicator dynamics as close as possible into an appropriate product market context. Practically, we concentrate our analysis on a specific segment of the German automobile industry where the different product types can be considered to be in competition with each other. Hence, we analyze the market for compact cars within the German automobile industry.

A shortcoming of this study might be that a dynamic perspective is not yet fully developed. In particular, we measure the competitive relation among products at a specific point in time by computing the corresponding fitness for each product. Then we explore the market share development over the subsequent years in order to answer the question of whether a car model's fitness indicator in the base year $t$ exerts influence on changes in market share between $t$ and a certain point in time $t+k$. By doing so, we implicitly assume that the characteristics of a car model remain the same over the whole time span between $t$ and $t+k$. This is a strong assumption. For instance, one could think of car producers reacting to the market performance of their products by changing the price or the quality characteristics (e.g., by face-lifting or offering supplementary equipment). We checked our data for price changes during the time spent in the market and found that the price of car models remains fairly stable. However, as we discussed earlier, our pricing information for new cars reflects list prices that do not incorporate temporary rebates or other price promoting methods. Concerning changes in quality attributes, we point out that our efficiency measure is based on the characteristics which are crucial for the purchasing decision. We can rule out that these characteristics undergo a fundamental change during the lifetime of a product.[17] For supplementary equipment or face-lifts this might not hold. We cannot deny that luxury or convenience features impinge on the

[17]In the case of fundamental modifications concerning the engine or the dimensions of a car, producers have to apply for a registration approval from the KBA. In such a case, the KBA records the modified car as a new model.

choice of consumers. However, if we conceive the consumer choice as a hierarchical elimination process as outlined by Devetag (1999), the reliance on the key characteristics can be justified. Nevertheless, implementing supplementary quality features in our analysis remains an important point on our research agenda.

Future work should aim at expanding the findings of this article in at least three directions. First, a more dynamic perspective across longer time spans will certainly provide a more accurate representation of the observed phenomena. Second, many additional insights might be gained by looking at different products and a dataset covering more periods. Third, in order to validate our results and to be able to derive more general conclusions, we have to uncover additional factors explaining the economic success of products and firms in an industry. The fairly small explanatory power of our model reveals that there is still considerable room for improvement in this respect. Nevertheless, the significant impact of the fitness indicator on changes in market shares shows that it is worth the effort to look at demand-side factors and product characteristics in order to explore the patterns of competitive dynamics on consumer goods markets.

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## Appendix A

### *A.1 Product evaluation using DEA*

Assume product quality of product $i$ at time $t$ is determined by a linear combination of $J$ product characteristics $q_{itj}\,(j=1,\ldots,J)$, collected together in a vector $\mathbf{q}_{it}=(q_{it1},\ldots,q_{itJ})'$, and denote the product price $p_{it}$ , the quality-price-ratio, $e_{it}$, can be stated as

$$e_{it}=\frac{a_1 q_{it1}+\ldots+a_J q_{itJ}}{p_{it}}=\frac{\mathbf{a}'\mathbf{q}_{it}}{bp_{it}},$$

where the vector $\mathbf{a}$ contains the weights for aggregating the product characteristics into the scalar product quality measure. The basic task is to compute the weights $\mathbf{a}$ in order to minimize the inverse fitness of product $i$, subject to a set of normalization restrictions, by solving the following fractional programming problem

$$\begin{aligned}\min_{\mathrm{a},b}\quad & \frac{1}{e_{it}}=\frac{bp_{it}}{\mathbf{a}'\mathbf{q}_{it}}\\ \text{s.t.}\quad & \frac{bp_{lt}}{\mathbf{a}'\mathbf{q}_{lt}}\geq 1\quad \forall l=1,\ldots,n\\ & \mathrm{a}>0\\ & b>0\end{aligned}$$

The weight $b$ in that program just serves as a normalizing constant and has no effect on the ability of the approach to compute suitable aggregation weights for the product characteristics (in effect the aggregation weights will just be equal to **a**/b). Charnes and Cooper (1962) have proposed a transformation into an ordinary linear programming problem that is straightforwardly solvable by the standard

**Table A.1** Correlation matrix for a subset of characteristics

| Variable | 1) | 2) | 3) | 4) | 5) | 6) | 7) | 8) | 9) | 10) | 11) | 12) | 13) | 14) | 15) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1) Torque | 1 | | | | | | | | | | | | | | |
| 2) Kilowatt | 0.57* | 1 | | | | | | | | | | | | | |
| 3) Engine Size | 0.69* | 0.60* | 1 | | | | | | | | | | | | |
| 4) Revolutions | −0.63* | −0.03 | −0.42* | 1 | | | | | | | | | | | |
| 5) Acceleration | −0.50 | −0.88* | −0.43* | −0.08 | 1 | | | | | | | | | | |
| 6) Speed | 0.60* | 0.95* | 0.58* | −0.06 | −0.88* | 1 | | | | | | | | | |
| 7) Fuel Consump. | −0.26* | 0.52* | 0.08 | 0.53* | −0.40* | 0.40* | 1 | | | | | | | | |
| 8) Fuel Capacity | 0.31* | 0.24* | 0.25* | −0.28* | −0.19* | 0.25* | −0.01 | 1 | | | | | | | |
| 9) Dimension | 0.28* | 0.22* | 0.24* | −0.13 | −0.08 | 0.23* | 0.07 | 0.14 | 1 | | | | | | |
| 10) Weight | 0.71* | 0.54* | 0.68* | −0.49* | −0.33* | 0.55* | 0.02 | 0.46* | 0.57* | 1 | | | | | |
| 11) Payload | 0.27* | 0.10 | 0.15* | −0.17* | −0.03 | 0.14 | −0.05 | 0.10 | 0.43* | 0.16* | 1 | | | | |
| 12) Airbag | 0.34* | 0.20* | 0.30* | −0.23* | −0.16* | 0.33* | −0.19* | 0.31* | 0.18* | 0.41* | 0.14* | 1 | | | |
| 13) Luggage | −0.04 | −0.08 | 0.050 | −0.02 | 0.16* | −0.01 | −0.03 | −0.03 | 0.56* | 0.21* | 0.09 | 0.03 | 1 | | |
| 14) Price | 0.78* | 0.75* | 0.65* | −0.38* | −0.57* | 0.76* | 0.14 | 0.33* | 0.44* | 0.75* | 0.34* | 0.41* | 0.04 | 1 | |
| 15) Fitness | 0.61* | 0.63* | 0.56* | −0.40* | −0.47* | 0.63* | −0.05 | 0.16* | 0.32* | 0.56* | 0.07 | 0.26* | 0.26* | 0.38* | 1 |

*Note*: *1% level of significance.

simplex algorithm. Performing the Charnes–Cooper transform gives the linear programming problem

$$\begin{aligned} \min_{\alpha,\beta} \quad & \frac{1}{e_{it}} = \beta p_{it} \\ \text{s.t.} \quad & \beta p_{lt} - \alpha' \mathbf{q}_{lt} \geq 0 \quad \forall l = 1, \ldots, n \\ & \alpha' \mathbf{q}_{it} = 1 \\ & \alpha > 0 \\ & \beta > 0 \end{aligned}$$

with the transformed weights $\alpha = \mathbf{a}/\mathbf{a}'\mathbf{q}_{it}$, $\beta = b/\mathbf{a}'\mathbf{q}_{it}$ and the additional normalization restriction $\alpha'\mathbf{q}_{it} = 1$.

Thus, the solution of the above linear program for each product and each time period gives a set of efficiency variables $e_{it}$ which result from a multilateral benchmarking performed by DEA. The inverse efficiency $1/e_{it}$ can be interpreted as the factor by which all characteristics of a product have to be increased in order to reach the efficiency level of the most efficient products in the sample (which get assigned a normalized efficiency value of unity).

For the actual computation of the fitness variable, one can take the dual of the above linear programming problem

$$\begin{aligned} \min_{\varphi,\lambda} \quad & \varphi \\ \text{s.t.} \quad & \mathbf{p}'_t \boldsymbol{\lambda} \leq p_{it} \\ \varphi \mathrm{q}_{it} - {} & \mathbf{Q}_t \boldsymbol{\lambda} \leq 0 \\ & \boldsymbol{\lambda} \geq 0 \end{aligned}$$

where $p_t = (p_{1t}, \ldots, p_{nt})'$ is the vector of prices in period $t$. The quality vectors of the $n$ products are collected together in the $J \times n$ matrix $\mathbf{Q_t} = (\mathbf{q}_{1t}, \ldots, \mathbf{q}_{nt})$. The solution values for $\boldsymbol{\lambda} = (\boldsymbol{\lambda}_1, \ldots, \lambda_n)'$ give the weights for the observations that serve as the benchmarks against which the efficiency is evaluated. The crucial feature of the duality theorem of linear programming which we exploit here is that the value of the target function at the optimum is unchanged. Thus, at the optimum, it holds that the inverse of the solution value for φ in the case of product $i$ at time $t$, $1/\varphi_{it}$, is equal to the efficiency variable $e_{it}$.

All the above reasoning implicitly rests on a restriction that is related to the assumption of constant returns to scale in an efficiency measurement application. To gain a more flexible benchmark, we have to introduce the additional constraint that the λ-values sum to unity, $\sum_{i=1}^{n} \lambda_i = 1$, which is analogous to the variables-returns-to-scale property in a production context.