

# Evolution in Markets

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1. Innovation and Selection/Competition are at the **core** of evolutionary approaches to markets/industries
2. **Formally**, extended versions of the **replicator dynamics framework** allow understanding the dynamic patterns of markets
3. **Empirically**, the validation of the theoretically derived pattern on the one hand but also of the general working of replicator dynamics is difficult and works „**occasionally**“
4. The low empirical evidence **challenges** the **replicator concept**

## Part I: Markets

## Part II: Theory of Markets as a locus of innovative change and competition

1. Selection dynamics
2. Selection and innovation dynamics
3. Selection, innovation and cooperation dynamics

## Part III: Empirics of Markets as a locus of innovative change and competition

1. Empirical approaches - an overview
2. Indirect measurement
3. Direct measurement

## Part IV: Problems in conception and measurement

Back-up: reading list, formal replicator framework, decomposition calculations

# Part I

## Markets

- **Markets are core to understanding economics**
  - Markets are considered to coordinate demand and supply
  - Markets are considered to organize competition
- **Major supply side dimensions of market**
  - Competition of firms
  - Innovation as major influence on competitiveness
- **Market dynamics**
  - The way demand and supply is coordinated over time (e.g. Cobb-web)
  - The way competition between firms develops over time
- **Market evolution**
  - The way innovation activities and market competition interact and shape the development of markets/industries (industrial dynamics, ILC, ...)

- “Romantic Ideas” on Innovation
  - innovative activities and the process of creative destruction (Schumpeter 1942) is based on a selection dynamics, implying that within a group of heterogeneous firms:
    - firms with innovative performance above average grow - or enter the market
    - firms with innovative performance below average shrink - eventually exit the market

- **Neo-Schumpeterian tradition (industrial dynamics)**
  - Based on Nelson/Winter (1982), .... Fagiolo/Dosi (2003)
  - micro behavior → industry outcomes → micro behavior → industry outcomes
  - dynamic forces: (1) technological competition and selection, (2) innovation
  - formal models: use of replicator (or Lotka-Volterra) equations (Metcalfe 1994)
- **Mainstream based heterogeneity models**
  - Jovanovic (1982), ... Aghion et al. (2005)
  - market competition: assumptions on the type of market, no dynamics
  - innovativeness is given by a stochastic process often combined with learning (imitation)
  - no connection between innovation, competition and reallocation
- **Industry Life Cycle models**
  - Klepper (1996)
  - endogenous determination of the conditions for entry and exit as well as for performance

## Part II

# Theory of Markets as a locus of innovative change and competition

1. Selection dynamics
2. Selection and innovation dynamics
3. Selection, innovation and cooperation dynamics

- Cantner U., Heterogenität, Technologischer Fortschritt und Spillover-Effekte, in: M. Lehmann-Waffenschmidt, Studien zur Evolutorischen Ökonomik V, Berlin: Duncker&Humblodt, 2002, 15-40
- Cantner U., Competition in innovation, in: U. Cantner, A. Greiner, T. Kuhn and A. Pyka (eds), Futurity and Economics, Edward Elgar: Cheltenham, 2009, 13-33.
- Cantner U., Evolution in Markets, mimeo, forthcoming in Pianta (2015), Proceedings of the LINCEI Conference, Roem Now. 2014

- Market competition via replicator dynamics

$$\dot{s}_i = s_i \cdot \lambda \cdot (f_i - \bar{f})$$

$$\forall i = 1, \dots, n \quad \bar{f} = \sum_j s_j f_j$$

- Firms / products indexed
- Market share
- Change of market share of firm  $i$  over time
- Fitness: unit costs, productivity, ...
- Speed of market selection
- In general all variables time indexed

$$i, i = 1, \dots, n$$

$$s_i \geq 0 \quad \sum s_i = 1$$

$$\dot{s}_i$$

$$f_i$$

$$\lambda > 0$$

- Dynamics depends on  $(f_i - \bar{f})$

$$(f_i - \bar{f}) > 0 \rightarrow \dot{s}_i < 0$$

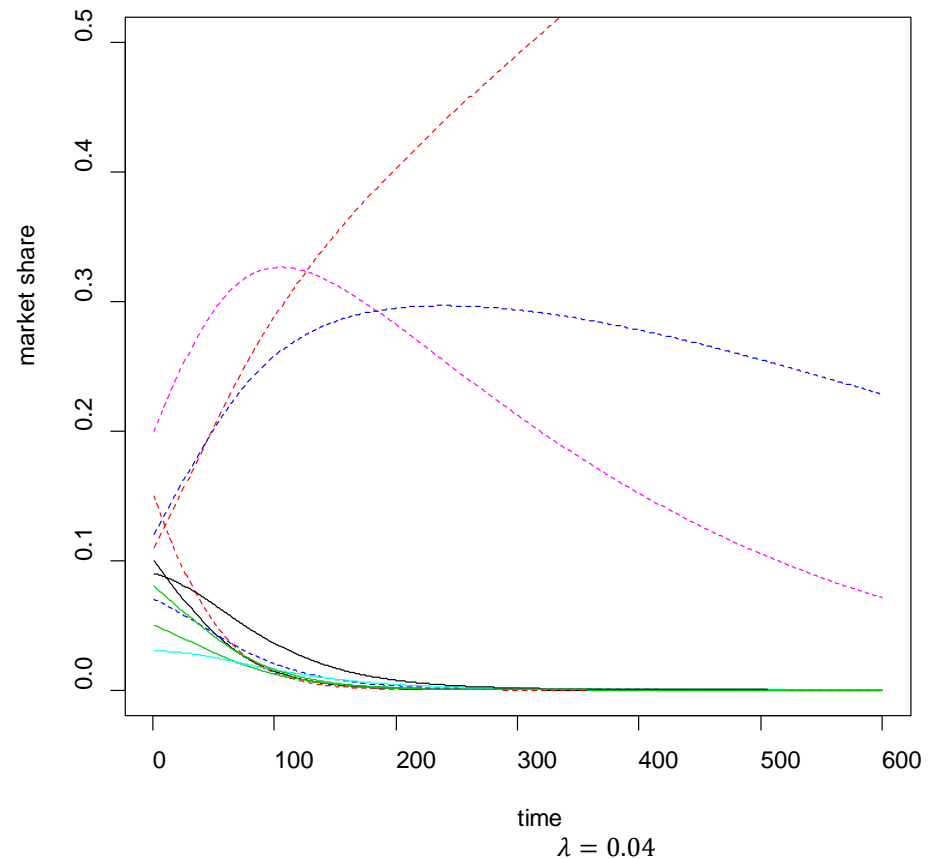
$$(f_i - \bar{f}) < 0 \rightarrow \dot{s}_i < 0$$

- Dynamics of  $\bar{f}$ :  $\dot{\bar{f}} \sim \sigma^2(f_i)$

- Market competition
  - Replicator dynamics

$$\dot{s}_i = s_i \cdot \lambda \cdot (f_i - \bar{f}) \quad \forall i = 1, \dots, n \quad \bar{f} = \sum_j s_j f_j$$

- Reducing initial heterogeneity
  - Market clearing
  - Differential profit
  - Differential (dis)investment



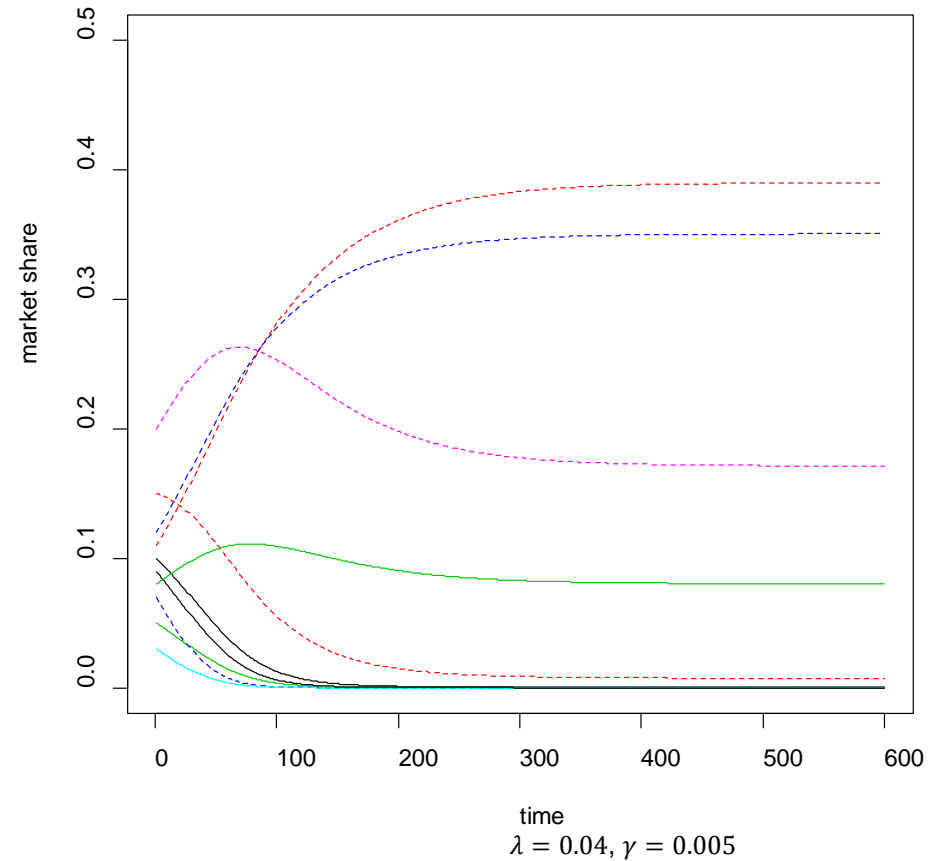
- Market competition  $\dot{s}_i = s_i \cdot \lambda \cdot (f_i - \bar{f}) \quad \forall i = 1, \dots, n \quad \bar{f} = \sum_j s_j f_j$
- Differential endogenous dynamics of heterogeneity
- Innovation  $\dot{f}_i = \gamma \cdot g(s_i) \quad [\rightarrow f_{max}]$ 
  - Case 1 - constant dynamic returns to scale:  $\partial \dot{f}_i / \partial s_i = 0 \quad \forall i = 1, \dots, n$
  - Case 2 - increasing dynamic returns to scale:  $\partial \dot{f}_i / \partial s_i > 0 \quad \forall i = 1, \dots, n$
  - Case 3 - decreasing dynamic returns to scale:  $\partial \dot{f}_i / \partial s_i < 0 \quad \forall i = 1, \dots, n$

## Case 1

constant dynamic returns to scale:

$$\partial \dot{f}_i / \partial s_i = 0 \quad \forall i = 1, \dots, n$$

Resulting pattern dependent on the relation between innovation dynamics  $\gamma$  and selection / competition dynamics  $\lambda$

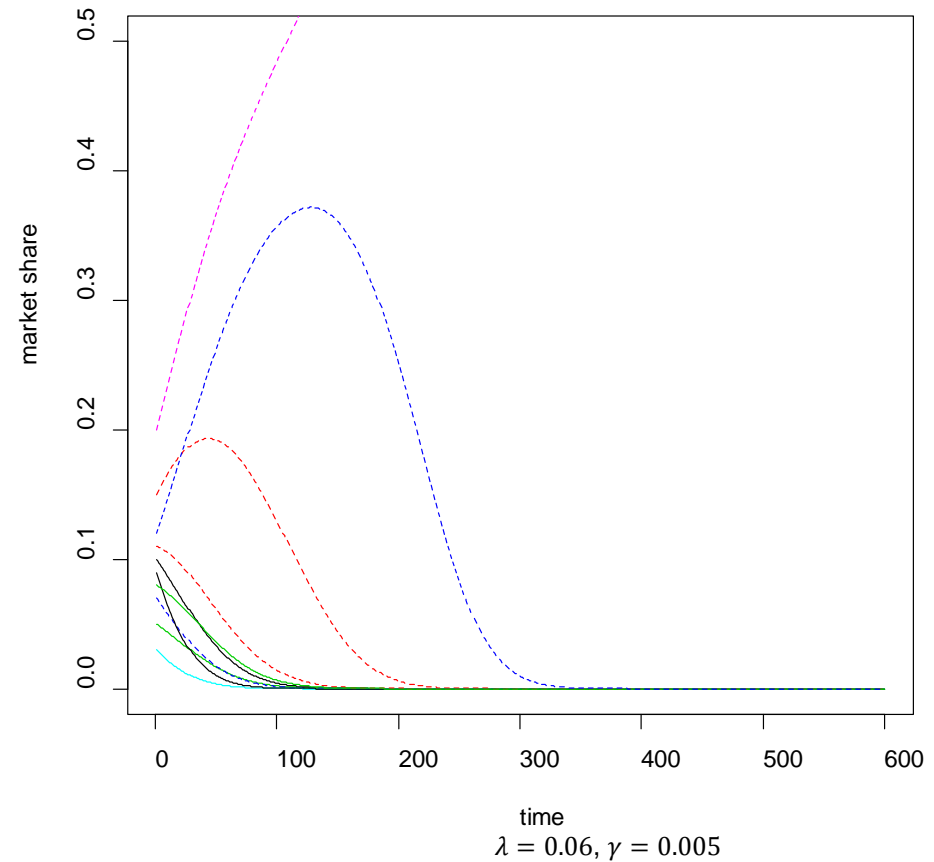


## Case 2

increasing dynamic returns to scale:

$$\partial \dot{f}_i / \partial s_i > 0 \quad \forall i = 1, \dots, n$$

Resulting pattern dependent on the relation between  $\gamma$  and  $\lambda$

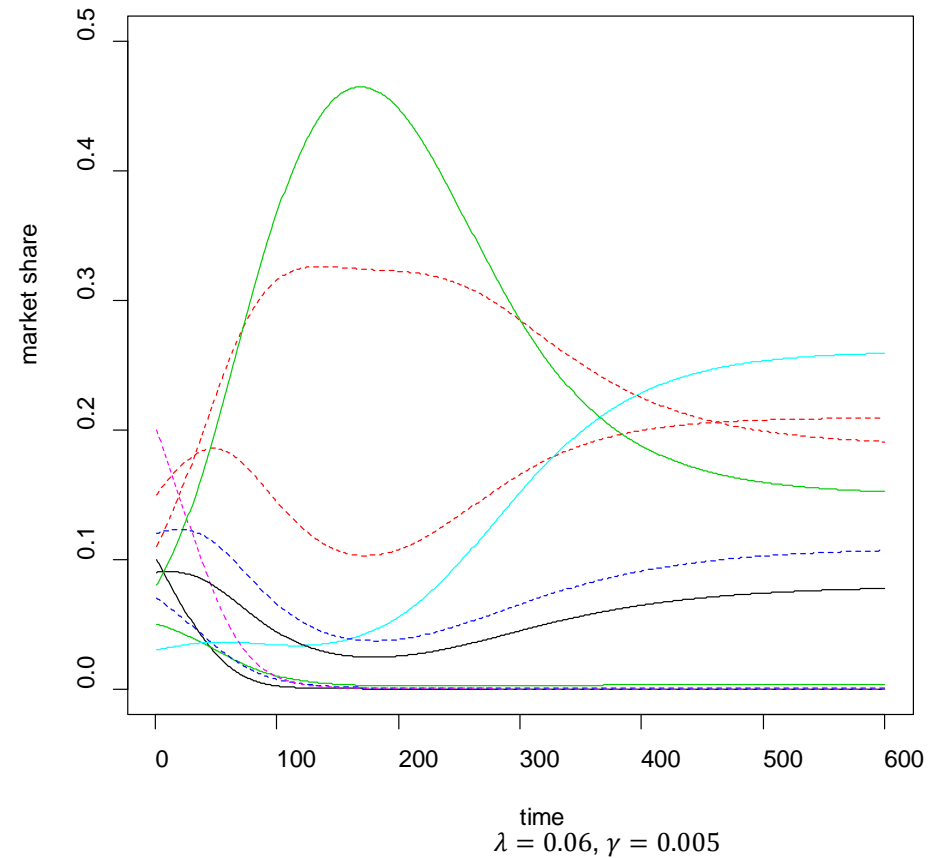


## Case 3

decreasing dynamic returns to scale:

$$\partial \dot{f}_i / \partial s_i < 0 \quad \forall i = 1, \dots, n$$

Resulting pattern dependent on the relation between  $\gamma$  and  $\lambda$



- Market competition

$$\dot{s}_i = s_i \cdot \lambda \cdot (f_i - \bar{f}) \quad \forall i = 1, \dots, n \quad \bar{f} = \sum_j s_j f_j$$

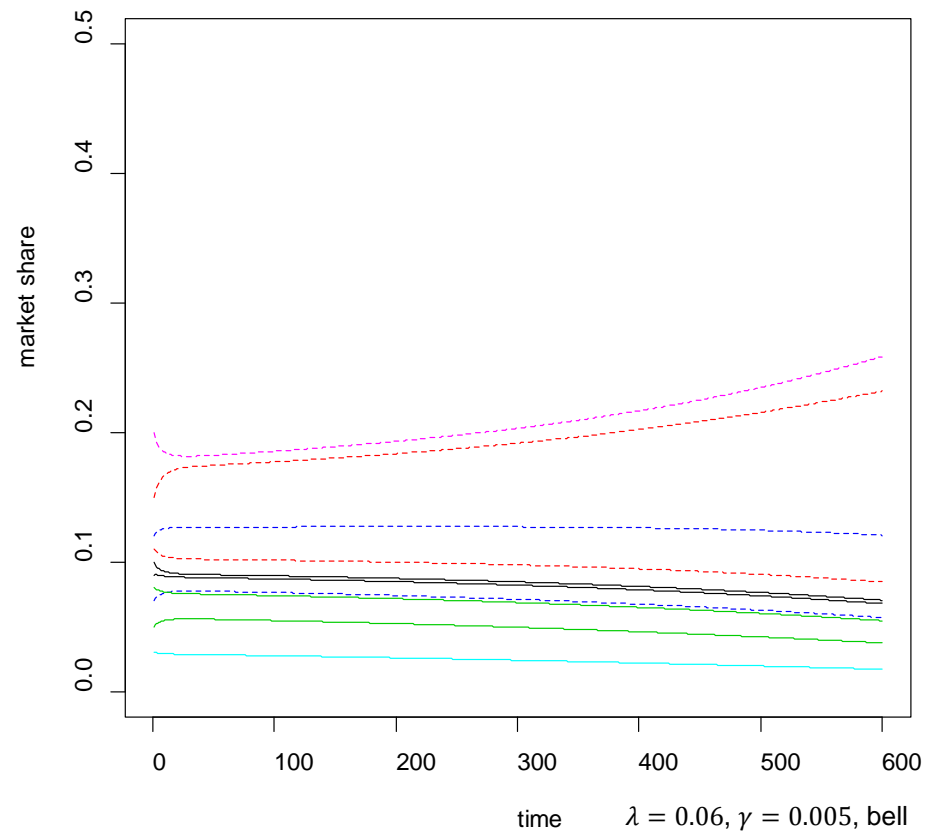
- Innovation

$$\dot{f}_i = \gamma \cdot g(s_i, \mathbf{z}_i) \quad [\rightarrow f_{max}]$$

- Another type of interaction

- Spillovers / Cooperation

$$\mathbf{z}_i = h(\max(f_j) - f_i)$$



## Part III

# Empirics of Markets as a locus of innovative change and competition

1. Empirical approaches - an overview
2. Indirect measurement
3. Aggregate performance and its decomposition
4. Direct measurement

## 1. Empirical Approaches - overview

### – Indirect approaches

- Coad (2007): balanced panel (France 1996-2004), profit rate as fitness, growth rates
- Bottazzi et al (2002, 2008): (Italy manufacturing 1998-2003), labor productivity, profitability as fitness, growth rates
- Cantner et al. (2009): ILC empirics, innovation related determinants of survival

### – Aggregate performance and its decomposition

- Baldwin & Gu (2003): for Canadian manufacturing; Disney et al. (2003): for UK manufacturing
- Metcalfe & Ramoglan (2007): decomposition of labor productivity and unit labor requirement
- Cantner & Krüger (2008): for German manufacturing
- Dosi et al (2015): comparing industries of various countries

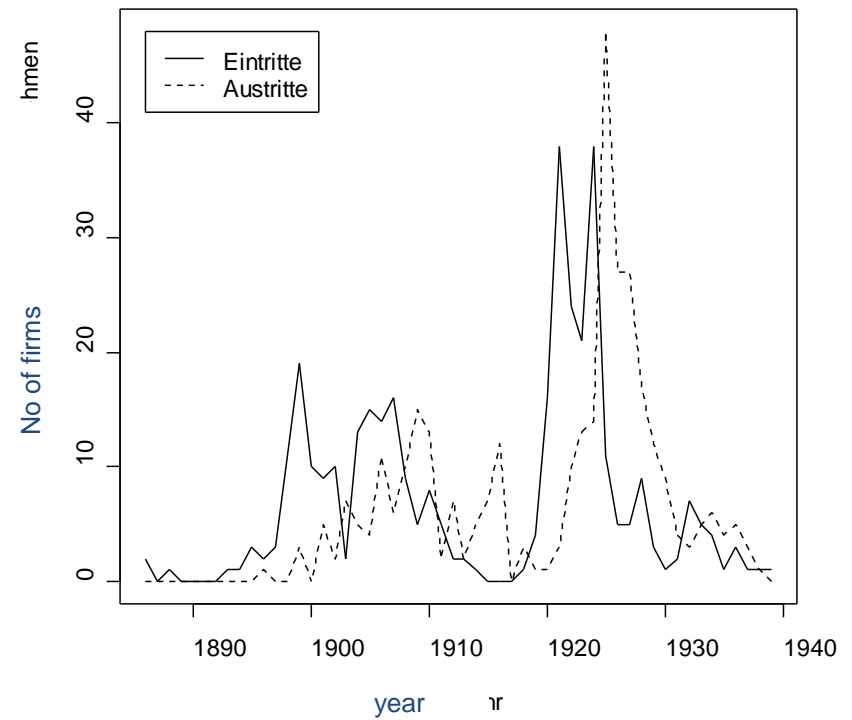
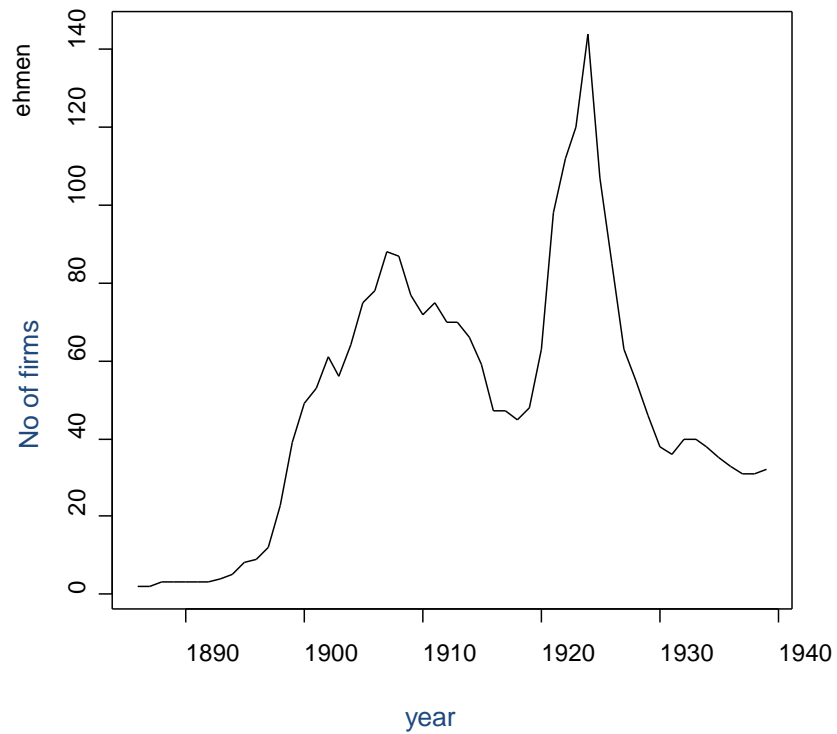
### – Direct approaches

- Metcalfe & Calderini (2000): unbalanced panel (Italy, steel, 1988-1996), unit costs as fitness, market share change, selection coefficient
- Cantner, Krüger & Söllner (2011): product level, market instead of industry demarcation, quality-price ratio as fitness criterion

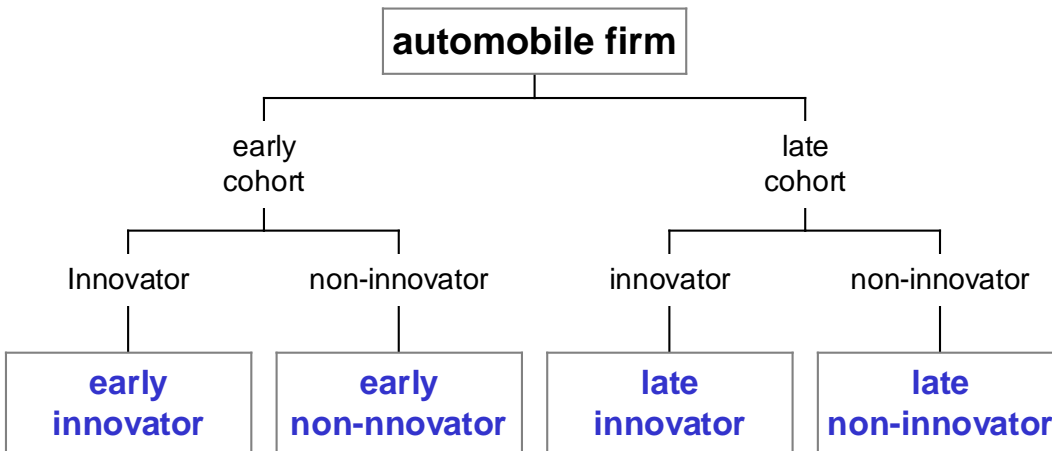
## 2. Indirect approaches

- Cantner, U., J. Krüger, K. v. Rhein, Knowledge Compensation in the German Automobile Industry, *Applied Economics* 43(22), 2011, 2941-2951

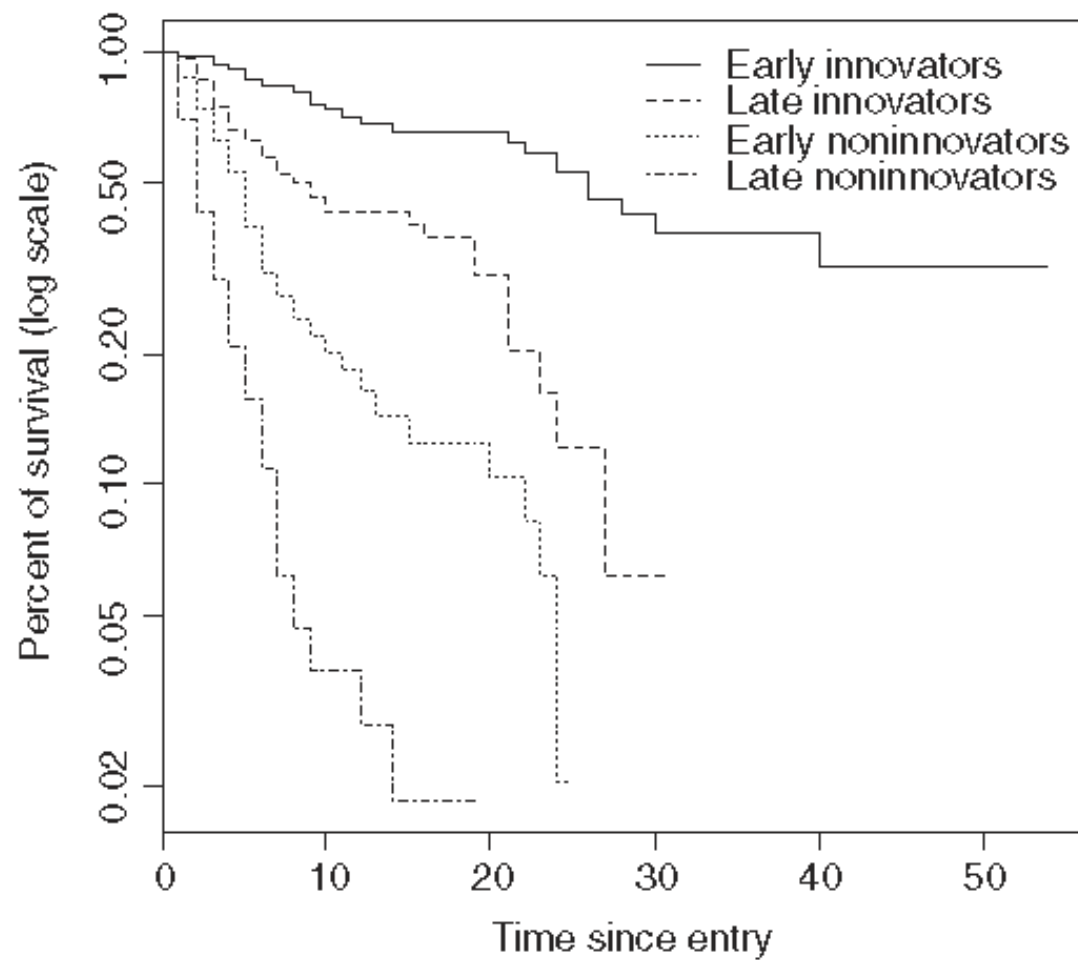
## German automobiles



(Source: Cantner/Dreßler 2003)



Category	Periode	# firms	variable
cohort 1	1886-1901	56	E1
cohort 2	1902-1906	52	E2
cohort 3	1907-1922	126	E3
cohort 4	1923-1939	115	E4



	(A)	(B)	(C)
Dep. variable: <b>exit hazard</b>	cohorts 1 vs. 2-4	cohorts 1-2 vs. 3-4	cohorts 1-3 vs. 4
early innovators	-2,224*** (0,002)	-1,956*** (0,001)	-2,170*** (0,000)
late innovators	-2,197*** (0,000)	-1,588*** (0,000)	-1,547*** (0,004)
late non- innovators	0,333 (0,399)	0,739*** (0,008)	0,760*** (0,000)
$R^2$	0,206	0,250	0,264
$n$	333	333	333

Notes: Robust standard are reported errors in parentheses. \*\*\*1%, \*\*5%, and \*10% level of significance

- Core results
  - Differential fitness in terms of innovativeness and in terms of accumulated experience show effect on survival
  - Exit dimension of replicator dynamics is taken into account
  - Market share dynamics not explicit
    - What kind of exit?
  - Very indirect test of replicator dynamics to work

### 3. Aggregate performance and its decomposition

- Cantner U., J. Krüger, Micro-Heterogeneity and Aggregate Productivity Development in the German Manufacturing Sector - Results from a Decomposition Exercise, Journal of Evolutionary Economics 18(2), 2008, 119-134

- Decomposition of the smooth development at the aggregate level into different underlying dynamics

- Productivity development at the industry level decomposed into firm level dynamics

- Market competition

$$\dot{s}_i = s_i \cdot \lambda(f_i - \bar{f}) \quad \forall i = 1, \dots, n \quad \bar{f} = \sum_j s_j f_j$$

- Innovation

$$\dot{f}_i = \lambda \cdot g(s_i) \quad [\rightarrow f_{max}]$$

- Aggregate Dynamics

$$\dot{\bar{f}} = \sum_j (\dot{f}_j s_j + \dot{s}_j f_j) \quad \text{within effect and between effect}$$

- Decomposition (Foster et al. 1998)

- Three groups of firms

- $C$ : persistent firms     $N$ : entering firms     $X$ : exiting firms

- Average productivity in  $t$        $\bar{a}_t^s = \sum_i s_{it} a_{it}$

- Average productivity in  $t - k$        $\bar{a}_{t-k}^s = \sum_i s_{it-k} a_{it-k}$

- Change in the average productivity between  $t - k$  and  $t$

$$\Delta \bar{a}_t^s = \bar{a}_t^s - \bar{a}_{t-k}^s = \sum_{i \in C \cup N} s_{it} a_{it} - \sum_{i \in C \cup X} s_{it-k} a_{it-k}$$

$$\Delta \bar{a}_t^s = \sum_{i \in C} s_{it-k} \Delta a_{it} + \sum_{i \in C} \Delta s_{it} (a_{it-k} - \bar{a}_{t-k}^s) + \sum_{i \in C} \Delta s_{it} \Delta a_{it} + \sum_{i \in N} s_{it} (a_{it} - \bar{a}_{t-k}^s) - \sum_{i \in X} s_{it-k} (a_{it-k} - \bar{a}_{t-k}^s)$$

within

between

covariance

entry

exit

Table 1  
Industry Composition of the Sample 1981-1998

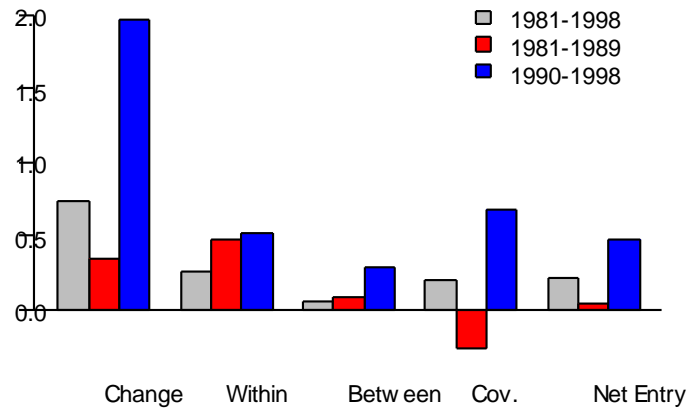
Industry	SIC2	Shortcut	Min. # Firms	Max. # Firms
Construction	15, 16, 17	Construction	22	49
Food and Beverages	20, 21	Food	53	87
Textiles and Apparel	22, 23	Textiles	26	48
Paper and Printing	26, 27	Paper	13	32
Chemicals and Petroleum	28, 29	Chemicals	50	107
Rubber and Plastics	30	Rubber	12	23
Metal Products	33, 34	Metal	45	91
Machinery and Equipment	35	Machinery	75	150
Electronics	36	Electronics	31	66
Transportation Equipment	37	Transportation	18	50
Instruments	38	Instruments	14	23

Table 2  
Foster-Haltiwanger-Krizan Decomposition 1981-98 (employment shares)

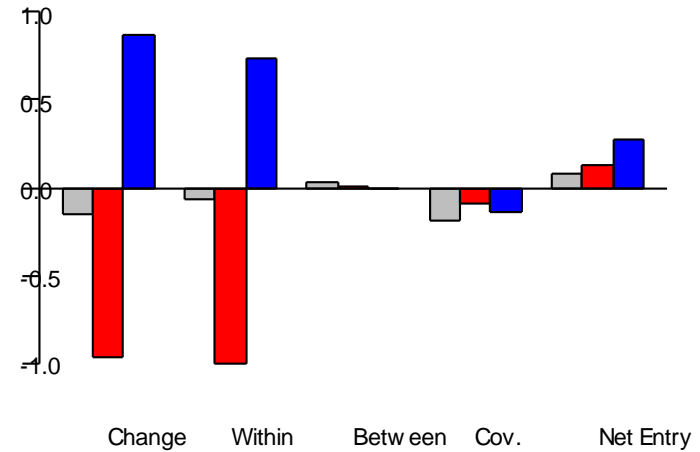
	Change	Within	Between	Cov.	Entry	Exit
Total Sample	0.7428	0.2654	0.0563	0.2066	0.1731	-0.0413
Construction	-0.1469	-0.0712	0.0298	-0.1902	0.0961	0.0114
Food and Beverages	0.1491	0.1195	-0.0344	0.1458	-0.0846	-0.0027
Textiles and Apparel	0.9975	0.5734	0.1571	-0.1970	0.3290	-0.1349
Paper and Printing	1.9066	0.3195	0.0649	-0.1263	1.7800	0.1314
Chemicals and Petroleum	0.9614	0.0705	0.2525	0.1770	0.3967	-0.0646
Rubber and Plastics	0.7528	0.5179	0.0099	0.3536	-0.0599	0.0688
Metal Products	0.2751	0.2112	0.0621	-0.0165	0.0717	0.0534
Machinery and Equipment	2.1975	0.5637	0.0790	0.9111	0.4631	-0.1805
Electronics	0.1253	-0.0322	0.2898	-0.1971	0.0190	-0.0458
Transportation Equipment	1.1350	0.5667	0.3450	0.0331	0.2198	0.0296
Instruments	0.9027	0.3495	0.0428	0.2455	0.2981	0.0333

Note: reported are average percentage growth rate of the aggregate productivity levels in the column change and the terms of the decomposition formula in the subsequent columns, each divided by the initial share-weighted average productivity level and multiplied by 100/(1998-1981).

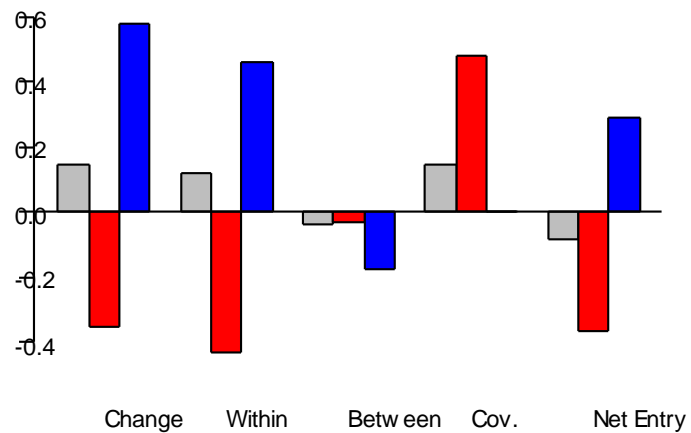
## Total Sample



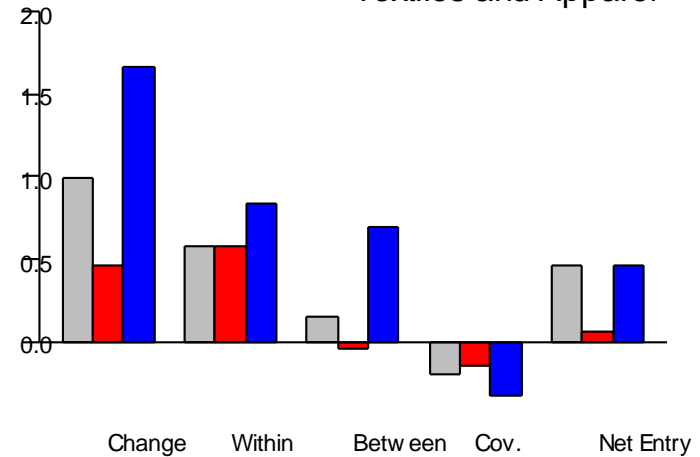
## Construction



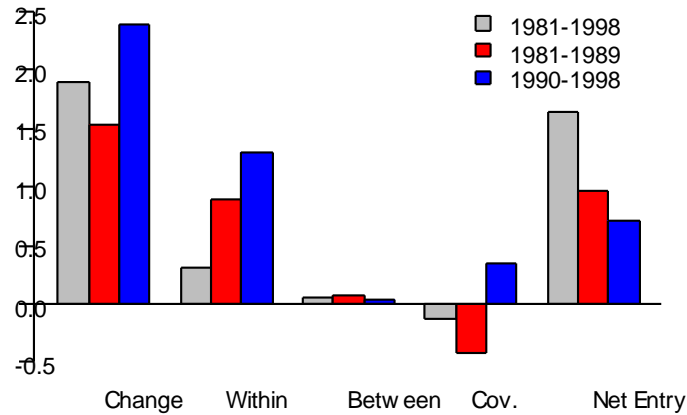
## Food and Beverages



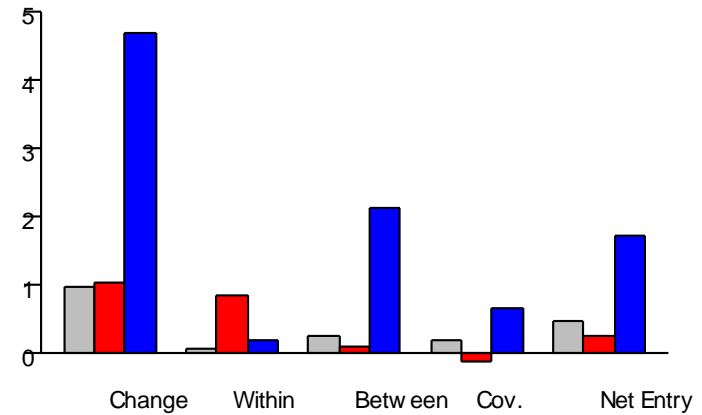
## Textiles and Apparel



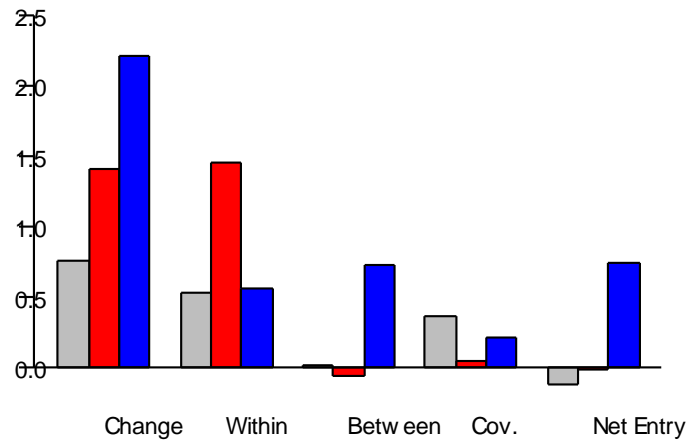
Paper and Printing



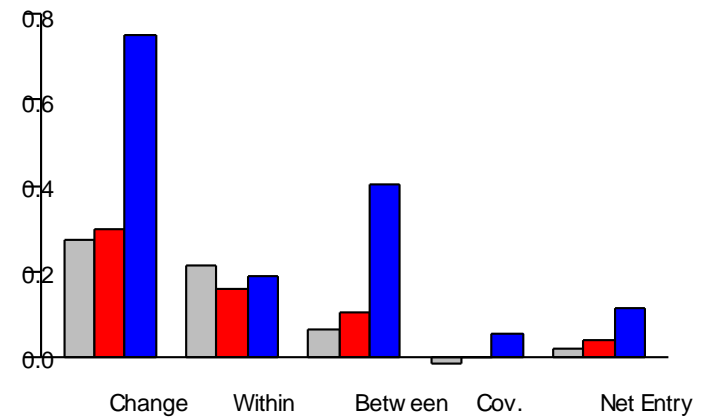
Chemicals and Petrole



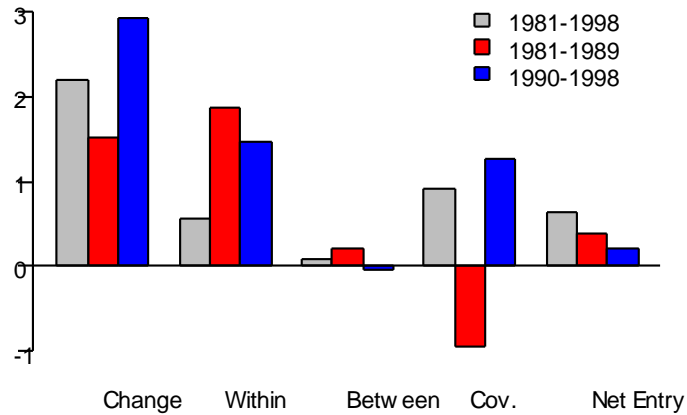
Rubber and Plastics



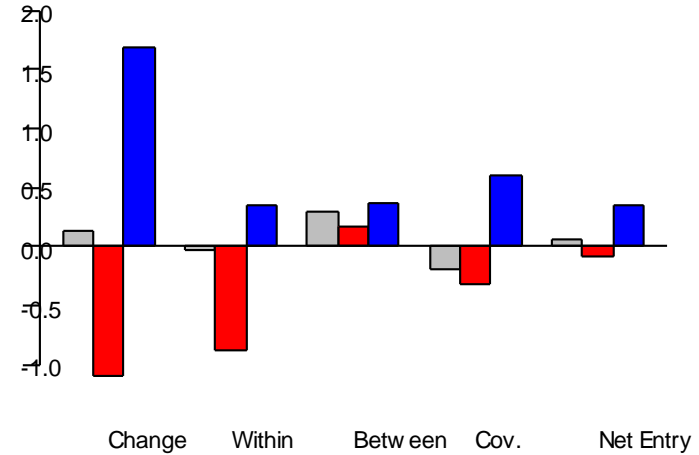
Metal Products



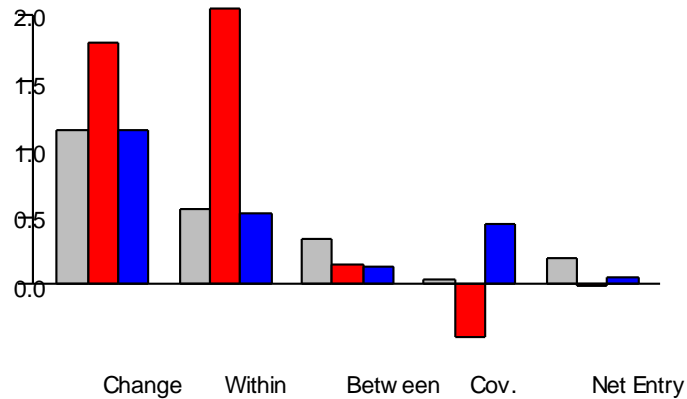
Machinery and Equipment



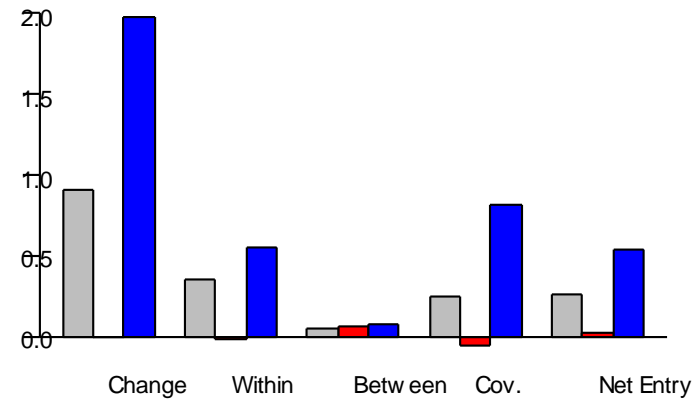
Electronics



Transportation Equipment



Instruments



- Core results

- Productivity growth **within** firms is essential part of aggregate productivity growth
- **Between** results seem to sustain the **mechanism of replicator dynamics** but have to be taken with care
- Entering firms tend to have a productivity level above average while exiting firms show a productivity level below average
- Success-breeds-success mechanism, coupling economical and technological improvements, take a non-negligible part of aggregate productivity development

## IV. Direct approaches

- Cantner U., J. Krüger, R. Söllner

Product Quality, Product Price and Share Dynamics in the German Compact Car Market

Industrial and Corporate Change 21(5), 2012, 1085-1115

- Not firms but **products** compete and become selected
- Fitness criterion for products: **quality-price ratio**

- $e$ : relative quality price ratio
- $i$ : product  $i$
- $t$ : time
- $q$ : characteristics 1 to  $J$
- $p$ : price
- $a, b$ : aggregation weights

$$e_{it} = \frac{a_1 q_{it1} + a_2 q_{it2} + \dots + a_J q_{itJ}}{p_{it}} = \frac{\mathbf{a}' \mathbf{q}_{it}}{b p_{it}}$$

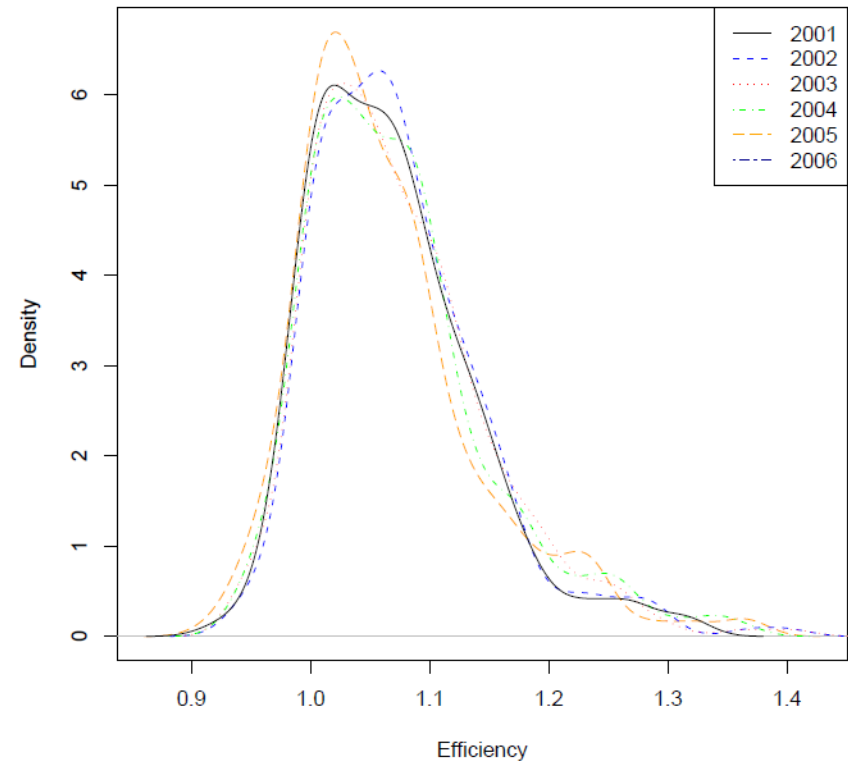
- Non-parametric frontier function approach (order- $m$ , order  $-\alpha$ )

- **Market demarcation** instead of industry demarcation

- **Sample**
  - Source: KBA
  - Period and Market: 2001-2006 German market for compact cars
- **Car characteristics**
  - Source ADAC
  - Out of 41 characteristics:
    - **performance of a car:** engine power (in kilowatts)
    - **environmental friendliness:** fuel efficiency via distance (in kilometers) per liter of petrol
    - **loading capacity:** luggage space (in liters)
    - **safety:** the dimension of a car (cubic meters)
- **Variables**
  - Fitness indicator **FIND**: share-weighted mean deviation of quality-price ratio
  - Age: age of the product
  - Firm dummies, price dummy, year dummies, country dummy

**Figure 3:** Kernel density estimates of car efficiency scores 2001-2005

Year	Min	1st Quan.	Median	Mean	3rd Quan.	Max
2001	0.919	1.017	1.059	1.068	1.107	1.323
2002	0.941	1.022	1.062	1.074	1.113	1.401
2003	0.947	1.020	1.061	1.074	1.110	1.405
2004	0.938	1.016	1.061	1.073	1.105	1.364
2005	0.913	1.011	1.048	1.066	1.095	1.375



- Theoretical model

$$\Delta s_{i,t+k} = s_{i,t+k} - s_{it} = \lambda s_{it}(e_{it} - \bar{e}_t)$$

$$\bar{e}_t = \sum_j s_{jt} e_{jt}$$

- Empirical model

$$\Delta s_{i,t+k} = \beta_1 FIND_{it} + \gamma' \mathbf{x}_{it} + u_{it}$$

$$FIND_{it} = s_{it}(e_{it} - \bar{e}_t)$$

$$\mathbf{x}_{it} = (Age_{it}, VW_i, Opel_i, \dots, Year\ Dum.)'$$

k=1	t=2001	t=2002	t=2003	t=2004	t=2005
FIND	-1.3813 (0.8603)	-0.1921 (0.72964)	10.1254** (4.2198)	-3.0383 (2.7386)	-1526 (3.3214)
R <sup>2</sup>	0.01864	0.3892	0.3255	0.05667	0.01718
Obs	326	286	300	313	336
k=2	t=2001	t=2002	t=2003	t=2004	
FIND	-2.6381** (1.1093)	10.6850** (5.3073)	13.4381** (5.0123)	9.2037** (3.7434)	
R <sup>2</sup>	0.03794	0.2551	0.1866	0.08587	
Obs	326	286	300	313	
k=3	t=2001	t=2002	t=2003		
FIND	6.9514 (6.4364)	12.9329** (5.1055)	10.6517* (5.5161)		
R <sup>2</sup>	0.08511	0.1642	0.06322		
Obs	326	286	300		
k=4	t=2001	t=2002			
FIND	9.7523* (5.3163)	14.6005** (7.3995)			
R <sup>2</sup>	0.0685	0.0831			
Obs	326	286			
k=5	t=2001				
FIND	8.4671 (5.7581)				
R <sup>2</sup>	0.0155				

	k=1	k=2	k=3	k=4
FIND	0.0521 (0.1199)	0.2870*** (0.0987)	0.3257*** (0.1005)	0.3392*** (0.1276)
Age	-0.1680*** (0.0276)	-0.0822** (0.0337)	-0.0822*** (0.0337)	-0.1363*** (0.0481)
VW	0.0390 (0.1001)	-0.0512 (0.0892)	0.0187 (0.1199)	0.1821 (0.1945)
Opel	-0.2653*** (0.0677)	-0.2634*** (0.0622)	-0.2945*** (0.0755)	-0.3644*** (0.0986)
Ford	0.0789 (0.0859)	-0.0824 (0.0948)	-0.0751 (0.0948)	-0.3713*** (0.1198)
Daimler	-0.2219 (0.2615)	0.1261 (0.3556)	0.0094 (0.4934)	-0.4910 (0.4317)
Audi	0.6282** (0.2588)	0.3184 (0.4032)	-0.2581** (0.1282)	-0.2969* (0.1787)
Toyota	-0.0260 (0.0903)	0.5071** (0.2182)	0.7460** (0.3590)	0.6744 (0.4695)
Skoda	0.2127*** (0.0682)	0.0496 (0.0720)	0.1777 (0.1186)	0.2978 (0.2012)
Citroen	0.0007 (0.0421)	0.0021 (0.0443)	0.0042 (0.0643)	0.0785 (0.1245)
Renault	-0.1937*** (0.0502)	-0.2111*** (0.0515)	-0.1560*** (0.0594)	-0.0789 (0.0552)
Peugot	0.0473 (0.0738)	0.2410* (0.1373)	0.2439 (0.1754)	0.4243* (0.2494)
Year Dummies	yes	yes	yes	yes
R <sup>2</sup>	0.053	0.118	0.132	0.122
F-statistic	5.64	5.27	5.87	5.13
p-value	0.000	0.000	0.000	0.000
Obs	1561	1225	912	612

	k=1	k=2	k=3	k=4
FIND	0.0658 (0.0981)	-0.0949 (0.1135)	-0.2404 (0.1515)	-0.2537 (0.1838)
German x FIND	0.0178 (0.1693)	0.4616*** (0.1613)	0.6645*** (0.1867)	0.6969*** (0.2292)
Year Dummies	yes	yes	yes	yes
R <sup>2</sup>	0.007	0.111	0.137	0.108
F-statistics	0.15	3.45	5.19	4.85
p-value	0.988	0.004	0.000	0.002
Obs	1561	1225	912	612

# Testing the selection mechanism for low-price and high-price cars

	Low-price	High-price
FIND	0.3477*** (0.1060)	0.3939 (0.3229)
Age	-0.1189** (0.0544)	-0.1645** (0.0825)
VW	0.0100 (0.4252)	0.2432 (0.1979)
Opel	-0.5570*** (0.1829)	-0.1974* (1117)
Ford	-0.3233*** (0.1101)	-0.5519 (0.3420)
Daimler	0.4853 (0.5376)	-0.8953* (0.5173)
Audi	- -	-0.2461 (0.2329)
Toyota	0.8617 (0.5801)	-0.1801 (0.1463)
Skoda	0.7965 (0.5269)	0.1333 (0.2220)
Citroen	0.0218 (0.1558)	0.1100 (0.1854)
Renault	-0.1239* (0.0639)	-0.1287 (0.1487)
Peugot	0.2787 (0.1972)	0.7008 (0.7347)
Year Dummies	yes	yes
R <sup>2</sup>	0.243	0.082
F-statistic	5.9	1.9
p-value	0.000	0.000
Obs	306	306

- Core results
  - Competition of products instead of firms  
→ car models
  - Market oriented approach instead of industry approach  
→ market for compact cars
  - Car models with considerably lower fitness than the market average lose market shares, while models with above-average fitness gain additional market shares  
→ evidence in favor of replicator dynamics
  - Even in case of various controls this result holds

## Part IV

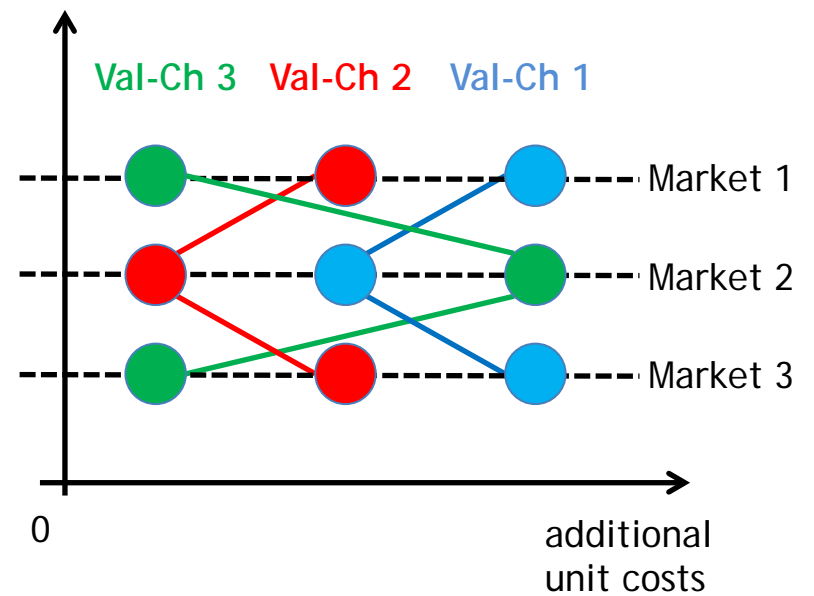
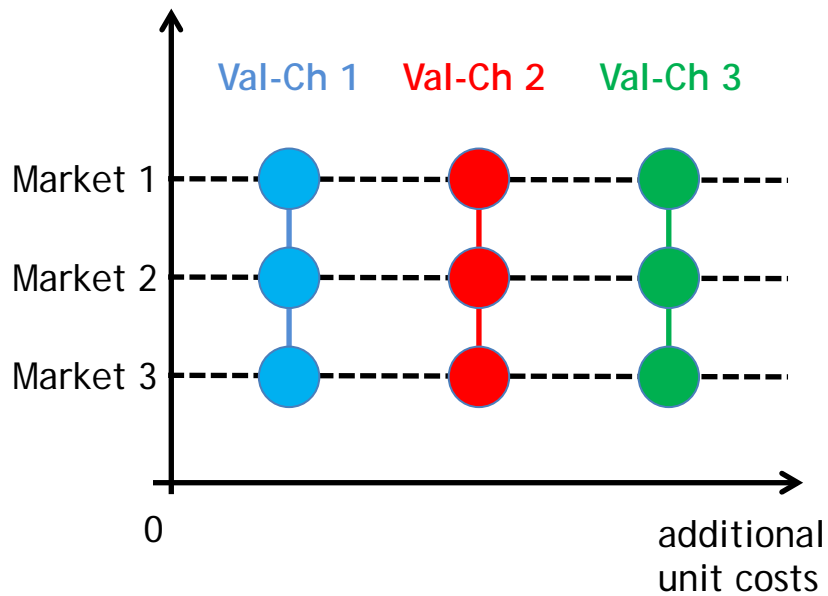
# Problems in conception and measurement

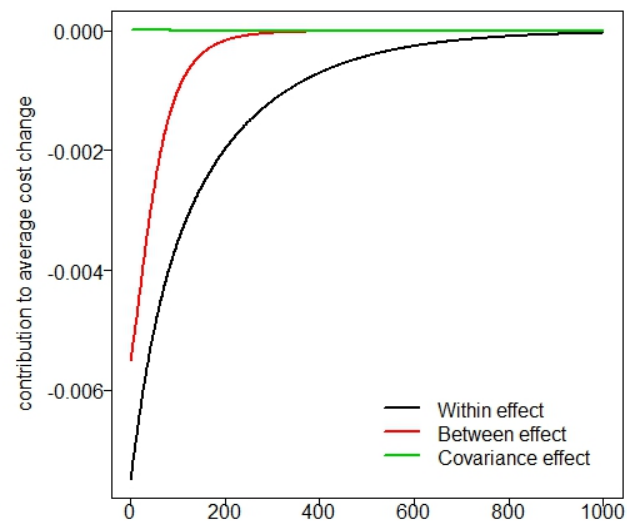
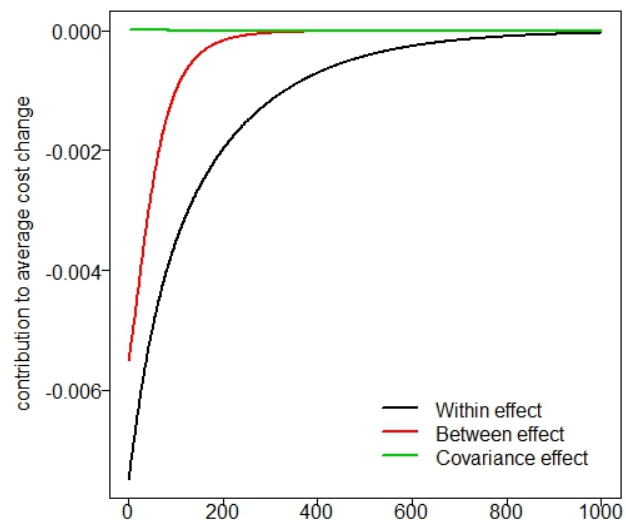
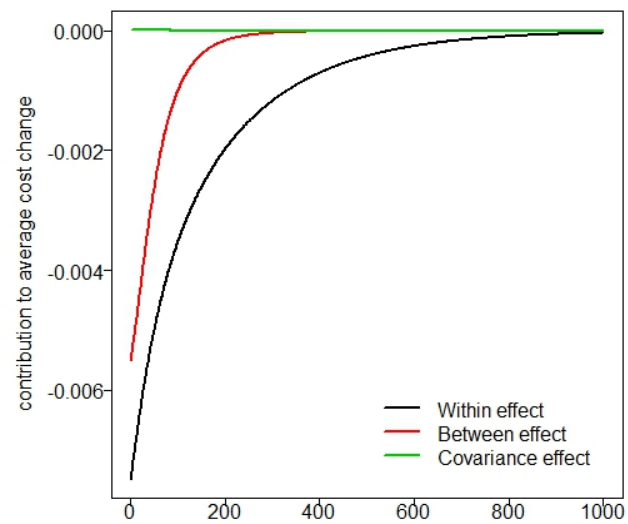
- Empirical evidence for selection dynamics is rather difficult to find
  - Replicator dynamics is simply an inappropriate analytical approach
  - Not all relationships among units of selection are taken into account
  - Variables used are not appropriate
  - Level of analysis is not appropriate

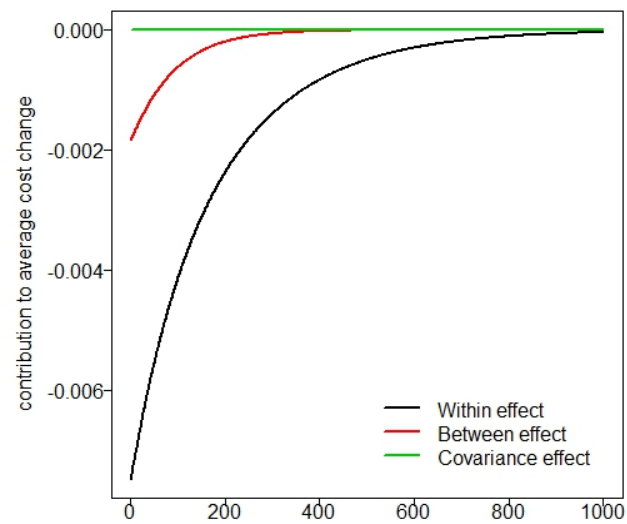
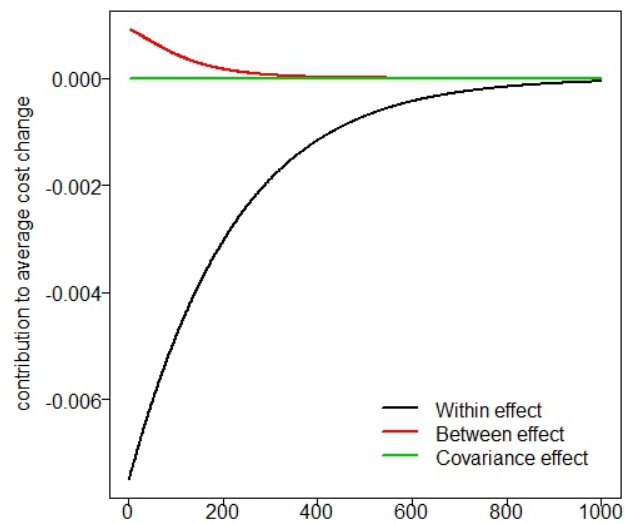
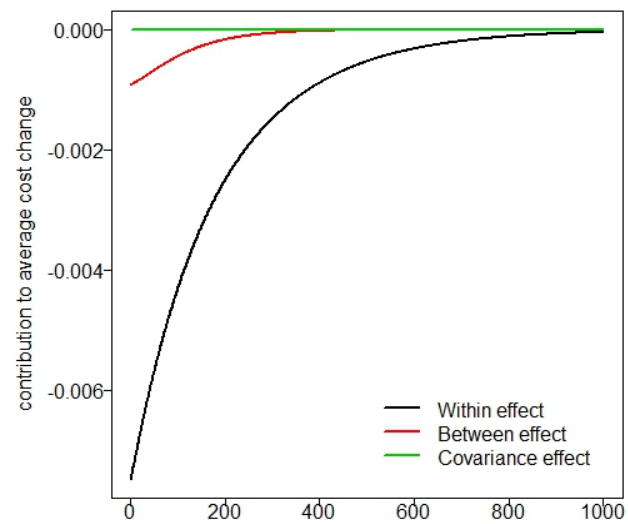
- Problems with the empirics

- Technological dynamics higher than economic dynamics (market shares → Cantner/Krüger 2006)
- Mobility reducing effects → mobility barriers
  - Market separation and niche markets (Klepper)
  - Sunk costs (Hölzl 2015)
- Intra-firm compensating effects
  - Multiproduct firms (cross subsidization)
- Inter-firm compensating effects - Interaction level
  - Collaboration
  - Connectivity via value chains

- Extending the model by value chain relationships
- Cantner, U., I. Savin and S. Vannuccini, Replicator dynamics in value chains: explaining some puzzles of market selection, Friedrich Schiller University Jena, 2015, mimeo







**There is more to be done!**

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**Thank you for your attention!!**

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# Back-up

- Baily, M.N., C. Hulten, D. Campbell (1992), Productivity Dynamics in Manufacturing Plants, Brookings Papers on Economic Activity: Microeconomics, 1992, 187-267.
- Baily, M.N., E.J. Bartelsman, J.C. Haltiwanger (1996), Downsizing and Productivity Growth: Myth or Reality?, Small Business Economics 8, 1996, 259-278.
- Bottazzi, G., A. Secchi, F. Tamagni (2008), Productivity, profitability and financial performance, Industrial and Corporate Change 17, 2008, 711-751.
- Cantner, U. (2002), Heterogenität, Technologischer Fortschritt und Spillover-Effekte, in: M. Lehmann-Waffenschmidt, Studien zur Evolutorischen Ökonomik V, Berlin: Duncker&Humblodt, 2002, 15-40
- Cantner, U. (2009), Competition in innovation, in: U. Cantner, A. Greiner, T. Kuhn and A. Pyka (eds), Futurity and Economics. Edward Elgar: Cheltenham, 2009, 13-33.
- Cantner, U., I. Savin, S. Vannuccini (2015), Replicator dynamics in value chains: explaining some puzzles of market selection, mimeo, Friedrich Schiller University Jena , 2015.
- Cantner, U., J. Krüger (2008), Micro-Heterogeneity and Aggregate Productivity Development in the German Manufacturing Sector - Results from a Decomposition Exercise, Journal of Evolutionary Economics 18(2), 2008, 119-134
- Cantner, U., J. Krüger, K. v. Rhein (2011), Knowledge Compensation in the German Automobile Industry, Applied Economics 43(22), 2011, 2941-2951
- Cantner, U., J. Krüger, R. Söllner (2012), Product Quality, Product Price and Share Dynamics in the German Compact Car Market, Industrial and Corporate Change 21(5), 2012, 1085-1115
- Coad, A. (2007), Testing the principle of growth of the fitter: The relationship between profits and firm growth, Structural Change and Economic Dynamics 18, 2007, 370-386.
- Coad, A. (2010), Exploring the processes of firm growth: evidence from a vector auto-regression, Industrial and Corporate Change 19, 2010, 1677-1703.
- Disney, R., J. Haskel, Y. Heden (2003b), Restructuring and Productivity Growth in UK Manufacturing, Economic Journal 103, 2003, 666-694.
- Dosi, G. (2007), Statistical Regularities in the Evolution of industries. A guide trough some evidence and challenges for the theory, in: Malerba, F. and Brusoni, S. (eds), Perspectives on innovation, Cambridge University Press, Cambridge, 2007, 153-186.

- Dosi, G., D. Moschella, E. Pugliese, F. Tamagni, (2015) Productivity, market selection and corporate growth: comparative evidence across US and Europe, Institute of Economics, Scuola Superiore Sant'Anna, Pisa, Italy, 2015
- Fisher, R. A. (1930), *The Genetical Theory of Natural Selection*. Clarendon Press: Oxford, 1930.
- Foster, L., J. Haltiwanger, C.J. Krizan (2001) Aggregate productivity growth: lessons from microeconomic evidence, in: Hulten, C. R., E. R. Dean, M. J. Harper (eds), *New Developments in Productivity Analysis*, University of Chicago Press: Chicago, 2001, 303-372
- Haltiwanger, J.C. (2000), *Aggregate Growth: What Have We Learned from Microeconomic Evidence?*, OECD Economics Department, 2000, Working Paper no. 267.
- Hölzl, W. (2015), Mobility barriers and the speed of market selection, *Journal of Evolutionary Economics* 25(2), 2015, 323-344.
- Krüger, J.J. (2014), Intra-sectoral structural change and aggregate productivity development - a robust stochastic non-parametric frontier function approach, *Empirical Economics* 46 (4), 2014, 1545-1572.
- Mazzucato, M. (1998), A computational model of economies of scale and market share instability, *Structural Change and Economic Dynamics* 9(1), 1998, 55–84.
- Mazzucato, M., W. Semmler (1999), Market share instability and financial dynamics during the industry life cycle: the U.S. automobile industry', *Special Issue of the Journal of Evolutionary Economics* 9(1), 1999, 67-96.
- Metcalf, J. S. (1998), *Evolutionary Economics and Creative Destruction*. Routledge: London, 1998.
- Metcalf, J.S. (1994), Competition, Fisher's Principle and Increasing Returns in the Selection Process', *Journal of Evolutionary Economics* 4(4), 1994, 321-46.
- Metcalf, J.S., M. Calderini (1998), Chance, necessity and competitive dynamics in the Italian Steel Industry, in: Cantner, U., H. Hanusch, S. Klepper (eds), *Economic Evolution, Learning, and Complexity*, Heidelberg, New York: Physica-Verlag, 1998, 139-158
- Olley, G.S., A. Pakes (1996), The dynamics of productivity in the telecommunications equipment industry, *Econometrica* 64, 1996, 1263-1297

– Derivation of selection equation:

- Market share of a firm:

$$s_i, \quad i \in \{1, \dots, n\}$$

$$s_i = y_i / y, \quad y = \sum_{j=1}^n y_j \quad \left[ \Rightarrow \dot{y} = \sum_{j=1}^n \dot{y}_j \right]$$

- Productive capacity of a firm  $y_i$
- Productive capacity of all firms  $y$

- Change of market share:  $\dot{s}_i = ds_i / dt$

$$\begin{aligned} \dot{s}_i &= \frac{\dot{y}_i}{y} - \frac{y_i}{y^2} \dot{y} = \frac{\dot{y}_i}{y} - \frac{y_i}{y} \sum_{j=1}^n \frac{\dot{y}_j}{y} = \frac{y_i}{y} \frac{\dot{y}_i}{y_i} - \frac{y_i}{y} \sum_{j=1}^n \frac{y_j}{y} \frac{\dot{y}_j}{y_j} \\ &= s_i g_i - s_i \cdot \sum_{j=1}^n s_j g_j = s_i \cdot (g_i - \bar{g}) \end{aligned}$$

- Growth rate of a firm  $g_i = \dot{y}_i / y_i$
- (market share weighted)  
average growth rate  $\bar{g}$

## – Investment routine:

- Unit profit
- Investment of a constant share
- Induced change production capacity

## – Insert in selection equation:

$$p - c_i$$

$$\lambda \in [0, 1]$$

$$g_i = \lambda \cdot (p - c_i)$$

$$\dot{s}_i = s_i \cdot \left( \lambda(p - c_i) - \sum_{j=1}^n s_j \cdot \lambda(p - c_j) \right)$$

$$= s_i \cdot \left( \lambda p - \lambda c_i - \lambda p + \lambda \cdot \sum_{j=1}^n s_j c_j \right)$$

$$= s_i \cdot (-\lambda c_i + \lambda \bar{c}) \quad \text{mit } \bar{c} = \sum_{j=1}^n s_j c_j$$

$$= s_i \cdot \lambda(\bar{c} - c_i)$$

$$f_i = -c_i$$

- result: selection equation  
with negative unit costs as fitness

- The change in average fitness in a population of competing firms is proportional to the variance in fitness

- Theorem:  $\dot{\bar{c}} = -\lambda \cdot \sigma_c^2 \leq 0$

- proof: 
$$\begin{aligned}\dot{\bar{c}} &= \frac{d\bar{c}}{dt} = \sum_{i=1}^n \dot{s}_i c_i = \sum_{i=1}^n s_i \cdot \lambda (\bar{c} - c_i) \cdot c_i \\ &= \lambda \cdot \sum_{i=1}^n (s_i c_i \bar{c} - s_i c_i^2) = -\lambda \cdot \sum_{i=1}^n (s_i c_i^2 - s_i c_i \bar{c} + (s_i c_i \bar{c} - s_i c_i \bar{c})) \\ &= -\lambda \cdot \left( \sum_{i=1}^n (s_i c_i^2 - 2s_i c_i \bar{c}) + \bar{c}^2 \right) = -\lambda \cdot \left( \sum_{i=1}^n s_i (c_i^2 - 2c_i \bar{c} + \bar{c}^2) \right) \\ &= -\lambda \cdot \left( \sum_{i=1}^n s_i (c_i - \bar{c})^2 \right) = -\lambda \cdot \sigma_c^2 \leq 0, \text{ da } \lambda > 0 \text{ and } \sigma_c^2 \geq 0 \quad \text{q.e.d.}\end{aligned}$$

- Internal consistency:  $\sum_{i=1}^n s_{it} = 1 \Rightarrow \sum_{i=1}^n s_{i,t+1} = 1$ 
  - continuous:  $\sum_{i=1}^n \dot{s}_i = \lambda \cdot \left( \sum_{i=1}^n s_i f_i - \sum_{i=1}^n s_{it} \cdot \bar{f} \right) = \lambda \cdot (\bar{f} - 1 \cdot \bar{f}) = 0$
  - discrete:  $\sum_{i=1}^n s_{i,t+1} = \sum_{i=1}^n s_{it} + \lambda \cdot \left( \sum_{i=1}^n s_{it} f_{it} - \sum_{i=1}^n s_{it} \cdot \bar{f}_t \right) = 1 + \lambda \cdot (\bar{f}_t - 1 \cdot \bar{f}_t) = 1$

$$\Delta \bar{a}_t^s = \bar{a}_t^s - \bar{a}_{t-k}^s = \sum_{i \in C \cup N} s_{it} a_{it} - \sum_{i \in C \cup X} s_{it-k} a_{it-k}$$

$$= \sum_{i \in C} (s_{it} a_{it} - s_{it-k} a_{it-k}) + \sum_{i \in N} s_{it} a_{it} - \sum_{i \in X} s_{it-k} a_{it-k}$$

$$= \sum_{i \in C} ((s_{it-k} + \Delta s_{it})(a_{it-k} + \Delta a_{it}) - s_{it-k} a_{it-k}) + \sum_{i \in N} s_{it} a_{it} - \sum_{i \in X} s_{it-k} a_{it-k}$$

$$= \sum_{i \in C} (s_{it-k} a_{it-k} + s_{it-k} \Delta a_{it} + \Delta s_{it} a_{it-k} + \Delta s_{it} \Delta a_{it} - s_{it-k} a_{it-k}) + \sum_{i \in N} s_{it} a_{it} - \sum_{i \in X} s_{it-k} a_{it-k}$$

$$= \sum_{i \in C} s_{it-k} \Delta a_{it} + \sum_{i \in C} \Delta s_{it} a_{it-k} + \sum_{i \in C} \Delta s_{it} \Delta a_{it} + \sum_{i \in N} s_{it} a_{it} - \sum_{i \in X} s_{it-k} a_{it-k} \quad + 0$$

$$= \sum_{i \in C} s_{it-k} \Delta a_{it} + \sum_{i \in C} \Delta s_{it} a_{it-k} + \sum_{i \in C} \Delta s_{it} \Delta a_{it} + \sum_{i \in N} s_{it} a_{it} - \sum_{i \in X} s_{it-k} a_{it-k} \quad + \sum_{i \in X} s_{it-k} \bar{a}_{t-k}^s - \sum_{i \in N} s_{it} \bar{a}_{t-k}^s - \sum_{i \in C} \Delta s_{it} \bar{a}_{t-k}^s$$

$$= \sum_{i \in C} s_{it-k} \Delta a_{it} + \sum_{i \in C} \Delta s_{it} a_{it-k} - \sum_{i \in C} \Delta s_{it} \bar{a}_{t-k}^s + \sum_{i \in C} \Delta s_{it} \Delta a_{it} + \sum_{i \in N} s_{it} a_{it} - \sum_{i \in N} s_{it} \bar{a}_{t-k}^s - \left( \sum_{i \in X} s_{it-k} a_{it-k} - \sum_{i \in X} s_{it-k} \bar{a}_{t-k}^s \right)$$

$$= \sum_{i \in C} s_{it-k} \Delta a_{it} + \sum_{i \in C} \Delta s_{it} (a_{it-k} - \bar{a}_{t-k}^s) + \sum_{i \in C} \Delta s_{it} \Delta a_{it} + \sum_{i \in N} s_{it} (a_{it} - \bar{a}_{t-k}^s) - \sum_{i \in X} s_{it-k} (a_{it-k} - \bar{a}_{t-k}^s)$$

$$\begin{aligned} \bar{a}_{t-k}^s (1-1) &= \bar{a}_{t-k}^s \left( \sum_{i \in C \cup X} s_{it-k} - \sum_{i \in C \cup N} s_{it} \right) = \\ \bar{a}_{t-k}^s \left( \sum_{i \in X} s_{it-k} + \sum_{i \in C} s_{it-k} - \sum_{i \in N} s_{it} - \sum_{i \in C} s_{it} \right) &= \\ \bar{a}_{t-k}^s \left( \sum_{i \in X} s_{it-k} - \sum_{i \in N} s_{it} - \sum_{i \in C} (s_{it} - s_{it-k}) \right) &= \\ \bar{a}_{t-k}^s \left( \sum_{i \in X} s_{it-k} - \sum_{i \in N} s_{it} - \sum_{i \in C} \Delta s_{it} \right) &= \\ \sum_{i \in X} s_{it-k} \bar{a}_{t-k}^s - \sum_{i \in N} s_{it} \bar{a}_{t-k}^s - \sum_{i \in C} \Delta s_{it} \bar{a}_{t-k}^s &= 0 \end{aligned}$$